Advanced Macroeconomic Methods

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Outline

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The model

The landlords

The landlords maximise

$$W_0^L = \underset{t=0}{\mathbb{E}} \sum_{t=0}^{\infty} \beta^t (logC_t^L + j \cdot logH_t^L - \frac{(L_t^L)^{1+\eta}}{1+\eta}) \qquad (1)$$

s.t.

$$C_t^L + q_t^h[(H_t^L - (1-\delta) \cdot H_{t-1}^L) + (H_t^Z - (1-\delta) \cdot H_{t-1}^Z)] + b_t$$
$$\leqslant q_t^Z \cdot Z_t + w_t^h \cdot L_{h_t}^L + w_t^c \cdot L_{c_t}^L + r_{t-1} \cdot b_{t-1} \qquad (2)$$

where C is consumption, H is housing, L is labour, q^h is housing price, H^L is owner occupied housing, H^Z is housing for rental purpose, b are real bonds, q^Z is the price of rental services, Z are rental services, w^h is wages in the housing sector, w^c is wages in the consumption sector, r is real interest rate, δ is the rate of depreciation of housing.

The model

The landlords

and

$$L_t^L = [\omega_l^{\frac{1}{\epsilon_l}} \cdot (L_{c_t}^L)^{\frac{1+\epsilon_l}{\epsilon l}} + (1-\omega_l)^{\frac{1}{\epsilon_l}} \cdot (L_{h_t}^L)^{\frac{1+\epsilon_l}{\epsilon l}}]^{\frac{\epsilon_l}{1+\epsilon_l}} \tag{3}$$

is an aggregate of labour in the different sectors. (see Rubio and Mora-Sanguinetti *Recent reforms in Spanish housing markets: An evaluation using a DSGE model* (2014)).

The model

The landlords

FOC: (substitute (4) into other FOCs)

$$\frac{\partial L}{\partial C_t^L} : \lambda_t = \frac{1}{C_t^L} \tag{4}$$

$$\frac{\partial L}{\partial b_t^L} : \frac{1}{C_t^L} = \beta^L \cdot \mathbb{E}_t[\frac{r_t}{C_{t+1}^L}] \qquad \text{Ee for consumption} \tag{5}$$

$$\frac{\partial L}{\partial H_t^L} : \frac{j}{H_t^L} = \frac{q_t^h}{C_{t+1}L} - \beta^L \cdot \mathbb{E}_t[(1-\delta) \cdot \frac{q_{t+1}^h}{C_{t+1}^L}] \tag{6}$$

The model

The landlords

FOC:

$$\frac{\partial L}{\partial L^L_{C_t}} : \frac{w^C_t}{C^L_t} = -(L^L_t)^\eta \cdot \omega_l^{\frac{1}{\epsilon_l}} \cdot (\frac{L^L_{C_t}}{L^L_t})^{\frac{1}{\epsilon_l}} \tag{7}$$

$$\frac{\partial L}{\partial L^L_{h_t}} : \frac{1-w^h_t}{C^L_t} = -(L^L_t)^\eta \cdot \omega_l^{\frac{1}{\epsilon_l}} \cdot (\frac{L^L_{h_t}}{L^L_t})^{\frac{1}{\epsilon_l}} \tag{8}$$

$$\frac{\partial L}{\partial H^Z_t} : \frac{q^h_t}{C^L_t} = \frac{A_{z_t} \cdot q^Z_t}{C^L_t} + \beta^L \mathop{\mathbb{E}}_t[(1-\delta) \cdot \frac{q^h_{t+1}}{C^L_{t+1}}] \tag{9}$$

The model

The tenants

The tenants maximise

$$W_0^T = \mathop{\mathbb{E}}_{t=0} \sum_{t=0}^{\infty} \widetilde{\beta}^t (log C_t^T + j \cdot log \widetilde{H}_t^T - \frac{(L_t^T)^{1+\eta}}{1+\eta}) \tag{10}$$

s.t.

$$\begin{split} C_t^T + q_t^h [H_t^T - (1-\delta) \cdot H_{t-1}^T] + q_t^Z \cdot Z_t + r_{t-1} \cdot b_{t-1} \\ \leqslant w_t^h \cdot L_{h_t}^T + w_t^c \cdot L_{c_t}^T + b_t \end{split} \tag{11}$$

The model

The tenants

where L_t^T is defined as for the landlords and

$$\widetilde{H}_t^T = [\omega_h^{\frac{1}{\epsilon_h}} \cdot (H_t^T)^{\frac{1+\epsilon_h}{\epsilon h}} + (1-\omega_h)^{\frac{1}{\epsilon_h}} \cdot (Z_t)^{\frac{1+\epsilon_h}{\epsilon h}}]^{\frac{\epsilon_h}{1+\epsilon_h}} \tag{12}$$

FOC:

$$\frac{\partial L}{\partial b}: \frac{1}{C_t^T} = \beta_T \cdot E\left[\frac{r_{t+1}}{C_{t+1}^T}\right] \tag{13}$$

$$\frac{\partial L}{\partial H_t^T}: \frac{j}{\widetilde{H}_t^T} \cdot (\omega_h)^{\frac{1}{\varepsilon_h}} \cdot \left(\frac{\widetilde{H}_t^T}{H_t^T}\right)^{\frac{1}{\varepsilon_h}} = \frac{q_t^h}{C_t^T} - E\left[\frac{(1-\delta)}{C_{t+1}^T} \cdot q_{t+1}^h\right] \tag{14}$$

$$\frac{\partial L}{\partial Z_t}: \frac{j}{\widetilde{H}_t^T} \cdot (1-\omega_h)^{\frac{1}{\varepsilon_h}} \left(\frac{\widetilde{H}_t^T}{a_t^Z \cdot H_t^Z}\right)^{\frac{1}{\varepsilon_h}} = \frac{q_t^Z}{C_t^T} \tag{15}$$

note that $Z_t = a_t^Z \cdot H_t^Z$ (rental service production function).

The model

The tenants

FOC:

$$\frac{\partial L}{\partial L_c^T} : \frac{w_t^c}{C_t^T} = \left(L_t^T\right)^{\eta} \cdot \omega_l^{\frac{1}{\varepsilon_l}} \cdot \left(\frac{L_{ct}^T}{L_t^T}\right)^{\frac{1}{\varepsilon_h}} \tag{16}$$

$$\frac{\partial L}{\partial L_c^T} : \frac{w_t^h}{C_t^T} = \left(L_t^T\right)^{\eta} \cdot (1 - \omega_l)^{\frac{1}{\varepsilon_l}} \cdot \left(\frac{L_{ht}^T}{L_t^T}\right)^{\frac{1}{\varepsilon_h}} \tag{17}$$

The model

The construction sector

The output in the construction sector is equal to the investment in housing *IH* and given through the production function

$$IH_t = L^T_{h_t} + L^L_{h_t} \tag{18}$$

We assume free entry and competition in the sector, which leads to zero profit or equivalently

$$q^h_t = w^h_t \tag{19}$$

Market clearing implies

$$H_t = IH_t + (1-\delta) \cdot H_{t-1} \tag{20}$$

The model

The consumption sector

The production function is assumed to be

$$Y_t = L^T_{c_c} + L^L_{c_t} \tag{21}$$

We assume free entry and competition in this sector as well and normalise the price for the consumption good to 1, which leads to zero profit or equivalently

$$1 =: p^c_t = w_{c_t} \tag{22}$$

We now report the model equation in the order they appear in our dynare code. In the landlord's section we include the aggregation of labour (3), the Euler equation for owner occupied housing (6), the log-linearised labour supply equations for the consumption and the housing sector (7) and (8). The next equation is the sum of the Euler equation for consumption (5) and the FOC w.r.t. rental services Z (9) (we combined them since they both represent a trade-off between consumption today and an investment in an asset in order to consume more in the following period). The last equation in the landlord's block is the landlord's budget constraint (2).

In the tenant's section of the model equations, we include the aggregation of labour (see landlord's specification) and the aggregation of housing (12). They are followed by the Euler equation for housing (14), the Euler equation for consumption (13), and the FOC wrt rental services (15). The labour supply in consumption and housing sector (16) and (17).

In the construction sector part of the model equations, the first equation is the production function for housing (18) combined with labour supply from landlords (8) and tenants (17) and solved for wages. It is followed by the free entry assumption (19) and the house market closing condition (20).

For the consumption sector, we only use the production function (21)

In the last model equation block we impose market clearing, AR(1) processes for technology in rental services A_Z as well as for the real interest rate r and three equations that yield aggregates for housing, labour and consumption (over landlord and tenant).