Steady State Example for Forum, from *Towards Simple Environmental Policy Rules: Fiscal Interactions, Time Inconsistency, and Externalities in General Equilibrium*

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The representative household intertemporally maximizes sequences of consumption, labor, and investment $\{C_t, L_t, K_{t+1}\}$ taking environmental quality as given

$$\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(C_t, L_t; S_t) \tag{1}$$

subject to their intertemporal budget constraint

$$C_t(1+\tau_c) + X_t(1+\tau_x) = w_t(1-L_t)(1-\tau_h) + (1-\tau_k)(r_t-\delta)K_t + \delta K_t + T_t \tag{2}$$

where leisure is net of hours worked

$$L_t = 1 - N_t \tag{3}$$

where q is the price of consumption and investment in terms of the numeraire gross output, $\tau = (\tau_c, \tau_x, \tau_k, \tau_d, \tau_h)$ is the set of government tax policies on consumption, investment, capital income, energy/pollution, and labor income, and r is the return on capital, $\delta \in (0,1)$ is the rate of depreciation, T is a lump sum rebate of tax revenues back to households such that public/private consumption are perfect substitutes.

Production.–There is a continuum of firms, indexed $i \in [0,1]$, that produces a homogeneous good that can be either consumed, invested to build capital, or used as an intermediate (energy) good in the production of other goods and services. All goods behave as perfect substitutes for consumption, investment, and intermediate goods. Each firm produces gross output, Y_i, using capital, labor, and the intermediate energy good purchased from other firms, E_i, given by a production function of the form

$$Y_{it} = \left[\chi_h \left(A_{kt} K_{it}^{\theta} N_{it}^{1-\theta}\right)^{\nu} + \chi_e (A_{et} E_{it})^{\nu}\right]^{1/\nu} \tag{4}$$

where θ is the relative share of capital, χ_h is the value share of the capital-labor composite versus energy, ν governs the elasticity of substitution between capital-labor and energy, and $A_{jt} = \exp(z_{jt})$ are corresponding shocks of the form

$$z_{jt} = \rho_j z_{j,t-1} + (1-\rho_j) g_j + \epsilon_{jt} \text{ for } j \in \{k, e\}$$

where $\epsilon_j \sim \mathcal{N}(0, \sigma_j^2)$ and g is the growth rate for capital-labor and energy technologies. Similar in theme to **??**, $\nu < 0$ measures the extent of the weak links in the economy. While intermediate goods are substitutable once created into output, they might be essential in the process of getting to the final output. For example, filling a car up with oil to visit a colleague might be a relative substitute compared to simply talking to the colleague via phone, but energy might still serve as a crucial ingredient in developing the phone in the first place. Under these assumptions about technology, firms maximize profits

$$\max \Pi = Y_i - wN_i - rK_i - E_i - \tau_d \xi E_i$$

where $\tau_d\xi$ is the emissions tax and the price of energy intermediates is normalized to the price of gross output so that the price of energy reduces to unity. Unlike **?** who treated energy as operating in infinite supply, energy as an intermediate good is produced endogenously, so its supply is limited by the amount that can be produced by capital and labor inputs.[1] Since all firms produce gross output using a common technology function, the price of intermediate enregy goods will equal th eprice of gross output such that firms are indifferent between deciding which firm to purchase energy from. The setup for production here contrasts with **?** who assumes perfect mobility for capital and labor between the final and intermediate goods sectors; here, treating energy as part of the representative firm in the economy highlights the role of distortions on intermediate goods.

Environmental Quality.–Energy produces emissions at a rate $\xi \in (0, 1)$ ("emissions intensity") that adversely affects environmental quality through the specification of a concentration response function given by

$$S_t = f(P_t; b) = \exp(b_0 - b_1 P_t) \tag{5}$$

where $P = E\xi$ denotes a measure of pollution (e.g., total suspended particulates) and f has the feature that $\partial S/\partial P < 0$ and $\partial S/\partial P > 0$. Lower air quality reduces the quality of private consumption and leisure. The function f is called a concentration response function (CRF). While log-linear CRFs have been widely implemented in the scientific literature (**??**), **?** highlights that the choice of functional form is still an active area of research in the natural sciences.

Government.–The government balances its budget each period; its revenues must equal its costs

$$T_t = \tau_h w_t(1 - L_t) + \tau_d \xi E_t + \tau_c C_t + \tau_x X_t + r_t \tau_k K_t \tag{6}$$

where the right hand side terms are, in the following order, tax revenues on labor income, carbon intensive energies, consumption, investment, and capital; the left hand side terms are tax revenues that are rebated back lump sum and used to offset distortionary labor income taxes, respectively.

0.1 Equilibrium Relationships

There are 10 endogenous variables consisting of $\{C_t, N_t, L_t, K_t, E_t, S_t, r_t, w_t, Y_t, V_t\}$, together with the distribution of shocks, that define the competitive equilibrium, formally defined below.

Definition 1. *The competitive equilibrium is a set of aggregate quantities for households, $\{C_t, N_t, K_{t+1}\}$, firms, $\{K_t, N_t, E_t\}$, market prices, $\{r_t, w_t\}$, and a distribution of shocks, $\{z_{kt}, z_{et}\}$, such that $\{C_t, N_t, K_{t+1}\}$ solve the Household Problem, $\{K_t, N_t, E_t\}$ solve the Producer Problem, the government balances its budget constraint, and markets clear at prices $\{r_t, w_t\}$ with output and value added given by Y_t and V_t.*

Household's problem.–Households choose $\{C_t, L_t, K_{t+1}\}$ in order to maximize utility subject to their budget and the environmental quality constraints. Taking the first order conditions and seeting equal to zero yields the the intertemporal and intratemporal Euler equations, respectively

$$u_C(C_t, L_t; S_t) = \beta \mathbb{E}_t \left(u_C(C_{t+1}, L_{t+1}; S_{t+1}) \left[1 - \delta + (1 - \tau_k) r_{t+1}/(1 + \tau_x)\right]\right) \tag{7}$$

$$\frac{u_L(C_t, L_t; S_t)}{u_C(C_t, L_t; S_t)} = w_t(1 - \tilde{\tau}) \tag{8}$$

where $(1 - \tau_h)(1 + \tau_x)/(1 + \tau_c) \equiv (1 - \tilde{\tau})$.

Producer's problem.–Since markets are perfectly competitive, and the price of the intermediate good is normalized to gross output, the factor prices are given by taking the following first order conditions of

$$\Pi = \left[\chi_k (A_{kt} K^\theta N^{1-\theta})^\nu + (1 + \tau_{dt}\xi) Y_t^{\nu-1} E_t\right]^{\frac{1}{\nu}} - r_t K_t - w_t N_t - E_t - \tau_{dt}\xi E_t$$

which yields the following equilibrium conditions

[1] I am grateful to Alessio Moro for making this observation. My treatment of intermediates as forming endogenously could be further microfounded by an expanding varieties model.

$$r_t = Y^{1-\nu} p A_k \chi_h (A_k K^\theta N^{1-\theta})^{\nu-1} \theta (K/N)^{\theta-1} \tag{9}$$

$$w_t = Y^{1-\nu} p A_k \chi_h (A_k K^\theta N^{1-\theta})^{\nu-1} (1-\theta)(K/N)^{\theta} \tag{10}$$

$$F_E \equiv Y^{1-\nu} A_e \chi_e (A_e E)^{\nu-1} = 1 + \tau_{dt} \tag{11}$$

Market clearing.–Market clearing requires that the time, environmental quality, government budget constraints hold (Equations 3, 5, and 6), value added is given by

$$V_t = Y_t - E_t$$

with an aggregate resource constraint

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t + E_t \tag{12}$$

To understand how environmental taxes affect output and the demand for energy, solve for τ_d in Equation 11 $\tau_{dt} = Y^{1-\nu} A_e \chi_e (A_e E)^{\nu-1} - 1$ and notice that

$$\frac{\partial \tau_{dt}}{\partial E_t} = F_{E,t}^{\nu-1} A_{et} \chi (A_{et} E_t)^{\nu-1} + Y_t^{1-\nu} A_{et}^2 \chi_e (\nu-1)(A_{et} E_t)^{\nu-2}$$

To the extent that $\nu \ll 0$, not only will $\partial \tau_d / \partial E < 0$, but also $\partial \tau_d / \partial K < 0$ and $\partial \tau_d / \partial N < 0$ since the tax directly affects the cost of energy, which affects the cost of capital and labor through complementarity.