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Forecasting with a DSGE Model of a Small Open Economy within the Monetary Union

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ABSTRACT
In this paper we lay out a two-region dynamic stochastic general equilibrium (DSGE) model of an open economy within the European Monetary Union. The model, which is built in the New Keynesian tradition, contains real and nominal rigidities such as habit formation in consumption, price and wage stickiness as well as rich stochastic structure. The framework also incorporates the theory of unemployment, small open economy aspects and a nominal interest rate that is set exogenously by the area-wide monetary authority. As an illustration, the model is estimated on Luxembourgish data. We evaluate the properties of the estimated model and assess its forecasting performance relative to reduced-form model such as vector autoregression (VAR). In addition, we study the empirical validity of the DSGE model restrictions by applying a DSGE-VAR approach. Copyright © 2014 John Wiley & Sons, Ltd.

KEY WORDS DSGE models; DSGE-VAR; open economy; forecasting; VAR

INTRODUCTION

In recent decades a new approach to macroeconomic modeling has involved the development of a generation of real business cycle models (the New Keynesian or New Neoclassical Synthesis models), which propose to extend the general equilibrium framework by introducing imperfect competition and nominal rigidities. An important feature of this class of models—often referred to as dynamic stochastic general equilibrium (DSGE)—is that monetary policy has a non-trivial effect on real variables. Therefore, studying the business cycle and macroeconomic implications of alternative government policies has been a natural application of this new generation of models and has motivated much research. Earlier contributions, including those which extend the framework to open economies, are Clarida *et al.* (1999, 2001), Benigno and Benigno (2003), Gali and Monacelli (2005) and many others. Recent developments in numerical and estimation methods enabled the application of advanced econometrics techniques to test the properties of the new generation of DSGE models, which showed better performance in capturing observed characteristics of real data due to stronger internal persistence mechanisms. Therefore, there is a growing interest from both academia and policy-making institutions in further advancing and using these models for studying macroeconomic fluctuations, assessing economic policy and forecasting. The most influential empirical papers in this area include Smets and Wouters (2003, 2007), who estimate a DSGE model similar in spirit to Christiano *et al.* (2005) for the euro area and USA respectively. The authors demonstrate that the estimated model provides a reasonable description of the economy and thus can serve as a useful tool for analysis of the effects of monetary policy and other structural shocks. Another important conclusion is that the forecasting performance of the DSGE model compares well with reduced-form structures such as vector autoregression (VAR) and Bayesian vector autoregression (BVAR) models. Following this seminal work, much research has been done to exploit DSGE modeling to study the macroeconomic fluctuations in various countries. In particular, Adolfson *et al.* (2008) examine the properties of a small open-economy model with modified uncovered interest parity (UIP) condition estimated on Swedish data. Lees *et al.* (2007) evaluate the performance of a small-scale DSGE model applied to New Zealand data. Lubik and Schorfheide (2007) estimate a small-scale DSGE model of a small open economy with a focus on the comparison of the monetary policy conduct in Australia, Canada, New Zealand and the UK. A number of studies employ a two-country framework to analyze the business cycle of European economies within the euro area. In particular, Pytlarczyk (2005) presents a DSGE model for Germany within the monetary union. Burriel *et al.* (2010) develop a DSGE model for the Spanish economy. There are also similar studies for Austria (Breuss and Rabitsch, 2009), France (Jondeau and Sahuc, 2004) and other countries.

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This paper contributes to the fast-growing DSGE literature described above and presents a model of a small open economy within the European Monetary Union, combining several of the features in the papers mentioned above. In particular, we develop a medium-scale two-region structural model with monopolistic competition in goods and labor markets. The model contains a number of frictions such as habit formation in consumption and price and wage rigidities, which became fairly standard in the recent literature. We adopt a small open economy setup that implies that the rest of the world (euro area) is not affected by domestic dynamics. As a result, the central bank policy instrument—the nominal interest rate—is exogenous from the home economy perspective. We derive a small open economy representation as a limiting case of a two-country framework and, unlike many of the recent DSGE papers, consider a medium- rather than small-scale specification with an explicit modeling of the labor markets and unemployment. In this respect, we follow an original paper by Gali *et al.* (2012; GSW hereafter) that incorporates unemployment into the Smets and Wouters (2007) closed-economy model.

From the empirical side, we contribute to the recent DSGE literature by presenting evidence for an additional country on the fit and forecasting performance of DSGE models estimated with a Bayesian approach. More specifically, we analyze the main properties of the estimated model, assessing the importance of various shocks and frictions for explaining the dynamics of the Luxembourgish economy.[1]

We then evaluate the model's point and density forecasting performance by comparing the accuracy of its out-of-sample predictions relative to those from reduced-form models such as VARs. In addition, we study the empirical validity of DSGE model restrictions by applying a DSGE-VAR analysis, as developed by Del Negro and Schorfheide (2004) and Del Negro *et al.* (2005). We include the DSGE-VAR model in the forecasting exercise in order to assess the ability of the DSGE-based versus atheoretical (BVAR) prior to improve the forecasting performance of the unrestricted VAR model.[2]

In the process of description of the estimation results we discuss how our work compares to previous studies. Our DSGE model shows a superior out-of-sample forecasting performance (at the 1-quarter-ahead horizon) compared to unrestricted VARs and BVARs. We also demonstrate that the restrictions implied by the DSGE model lead to an improvement of the performance of the standard VAR in predicting dynamics of the labor market variables such as wages and unemployment.

The paper is organized as follows. In the next two sections we present our small open-economy model and its log-linear representation. The fourth section describes the data, alternative forecasting models and estimation results. The forecast evaluation and comparison are presented in the fifth section. Finally, the sixth section contains some concluding remarks.

A SMALL OPEN-ECONOMY MODEL

In this section we formulate an open-economy DSGE model with theoretical foundations closely related to the papers by Gali and Monacelli (2005) and De Paoli (2009).[3] The model contains a number of rigidities typically used in the empirical DSGE literature in order to capture the properties of real data (Christiano *et al.*, 2005; Smets and Wouters, 2003, 2007). In particular, we introduce habit formation in consumption as well as Calvo price and wage stickiness. Moreover, we explicitly incorporate the theory of unemployment into the model setup following the recent paper by GSW.

The framework is represented by a two-country dynamic general equilibrium model where both sides, Home (the small open economy: H) and Foreign (the rest of the world, the relatively closed economy: F), are explicitly modeled. A continuum of infinitively lived domestic households belongs to the interval $[0, n)$, while foreign agents belong to the segment $(n, 1]$. The small open-economy problem is derived as a limiting case ($n \to 0$) of such a framework (as in De Paoli, 2009). Therefore, the home economy, owing to its small size, is assumed to have a negligible impact on the rest of the world. Households receive utility from consumption and disutility from work. The home economy is composed of final and intermediate goods producers, consumers and labor unions.[4] Agents consume

[1] As for existing structural models for Luxembourg, Pierrard and Sneessens (2009) have developed an overlapping generations (OLG) small open-economy model. The authors concentrate on modeling the realistic features of the Luxembourg labor market. The 'pure' OLG representation allows study of demographic questions such as the consequences of the aging of the population and the potential effects of alternative macroeconomic policies. The model is then calibrated on Luxembourg data and simulated. Other studies for Luxembourg based on the DSGE methodology include papers by Deak *et al.* (2011, 2012). These papers present an LSM—DSGE small open-economy model for Luxembourg—which is built following Blanchard's (1985) OLG approach. The model incorporates more realistic goods market structure with monopolistic competition, the distinction between tradable, non-tradable goods and the banking sector. The model is calibrated and used to study the reaction of the economy to real and financial shocks.

[2] The working paper version of this work, Marcellino and Rychalovska (2012), also contains the analysis of contribution of structural shocks to business cycle fluctuations. In particular, we use the estimated model to calculate variance decompositions and impulse responses, in order to evaluate the sources and propagation of macroeconomic fluctuations.

[3] We focus on the main model equations relevant for the open-economy specification. The rest of the model is rather standard. More detailed derivation of the structural equations can be found in Marcellino and Rychalovska (2012).

[4] We assume a somewhat simplified structure for the foreign economy. In particular, we abstract from explicit modeling the production side and assume that households are both consumers and producers. Moreover, we assume that there are no labor market frictions and unemployment.

the final consumption good, which includes goods produced by the domestic economy as well as imported goods. The share of imported goods may vary in the consumption basket of each country. Thus the model allows for the presence of home bias in consumption. Firms, which are monopolistically competitive, hire labor to produce differentiated goods. Prices on the goods market are assumed to be sticky and evolve according to Calvo's staggering scheme (1983). In addition, we assume monopolistic competition and Calvo wage-setting behavior on the labor market. Furthermore, production subsidies are introduced in order to offset the monopolistic distortions. In this version of the model, we abstract from capital accumulation. The international and domestic asset markets are complete. The law of one price holds for individual goods at all times. The small open economy is assumed to belong to the common currency area with the foreign country. The monetary authority (ECB) sets the interest rate following the Taylor rule, based on the economic performance of the whole EMU. Thus the interest rate is an exogenous variable from the small open-economy perspective.

Representative households and preferences

The expected lifetime utility function maximized by a representative household of country H is given by

$$U_t^j = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \varepsilon_t^c \left[U\left(\tilde{C}_t^j\right) - \varepsilon_t^l V\left(L_t^j\right)\right]\right\} \tag{1}$$

where j is the index specific to the household; $\tilde{C}_t^j$ denotes the time t per capita consumption of the composite commodity bundle, L_t^j is the labor effort and $0 < \beta < 1$ is the intertemporal discount factor. There exists a continuum h of different labor types, denoted by $l_t^j(h)$ and indexed for home country on the interval $[0, n]$. Then labor effort of the individual j is defined as $L_t^j = \int_0^n l_t^j(h)\mathrm{d}h$. ε_t^c and ε_t^l denote an exogenous preference and labor supply shocks, respectively. In our analysis we assume that preferences have the following functional form:

$$U\left(\tilde{C}_t^j\right) = \frac{\left(C_t^j - \chi C_{t-1}\right)^{1-\sigma_c}}{1-\sigma_c}, \quad V\left(L_t^j\right) = \frac{\left(L_t^j\right)^{1+\eta}}{1+\eta}$$

where $\sigma_c > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, and $\eta \geq 0$ is equivalent to the inverse of the elasticity of labor supply. χ is an external habit formation parameter, which determines the dependence of the current individual consumption from the aggregate lagged consumption index. The composite consumption good C is a Dixit–Stiglitz aggregator of goods produced at home and abroad and defined as

$$C^j = \left[\upsilon^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-\upsilon)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{2}$$

Preferences for the rest of the world (denoted by an asterisk) are specified in a similar fashion:

$$C^{j*} = \left[(\upsilon^*)^{\frac{1}{\theta}} \left(C_H^*\right)^{\frac{\theta-1}{\theta}} + (1-\upsilon^*)^{\frac{1}{\theta}} \left(C_F^*\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{3}$$

where $\theta > 0$ is the intratemporal elasticity of substitution, and υ and υ^* are the parameters that determine the preferences of agents in countries H and F, respectively, for the consumption of goods produced at Home. As in Sutherland (2002) and De Paoli (2009) we assume that $(1-\upsilon)$, the share of imported goods from country F in the consumption basket of country H, increases proportionally to the relative size of the foreign economy $(1-n)$ and the degree of openness α. Therefore, $(1-\upsilon) = (1-n)\cdot\alpha$. Similarly, $\upsilon^* = n\cdot\alpha$. Such a specification allows modeling of home bias in consumption as a consequence of different country size and degree of openness.

The consumption sub-indices of home and foreign-produced goods C_H and C_F are composed of differentiated goods $c_h(z)$ and $c_f(z)$ with the elasticity of substitution across the differentiated goods $\sigma > 1$. The solution to the cost minimization problem yields the following demand equations for differentiated goods produced at home and abroad:

$$c_h(z) = \frac{1}{n}\left(\frac{p_h(z)}{P_H}\right)^{-\sigma} C_H, \quad c_f(z) = \frac{1}{1-n}\left(\frac{p_f(z)}{P_F}\right)^{-\sigma} C_F \tag{4}$$

where $p_H(z)$ and $p_F(z)$ are prices (in units of the domestic currency) of the home-produced and foreign-produced intermediate goods. P_H is the domestic price index and P_F is a price index for goods imported from country F. The price indices represent cost-minimizing prices of a unit of final (home or foreign) good basket.

Furthermore, optimal allocation of expenditures between domestic and imported goods is given by

$$C_H = \upsilon \left(\frac{P_H}{P}\right)^{-\theta} C, \quad C_F = (1-\upsilon)\left(\frac{P_F}{P}\right)^{-\theta} C \tag{5}$$

where

$$P = \left[\upsilon P_H^{1-\theta} + (1-\upsilon) P_F^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{6}$$

is the consumer price index for country H.

Similar demand functions can be derived for the foreign country.

Asset market structure and consumer's problem

Similar to Chari *et al.* (2002) we assume that foreign and domestic households have access to the international financial market, where state-contingent nominal bonds denominated in the home currency are traded. Thus, markets are complete domestically and internationally. The budget constraint of the consumer in the home country at period t is given by

$$P_t C_t^j + B_{t+1}^j / R_{t+1} \leq B_t^j + W_t^j L_t^j + TR_t \tag{7}$$

where B_{t+1}^j is the holding of a nominal state-contingent bond that pays one unit of home currency in period $t+1$, R is the gross nominal interest rate, $W_t^j L_t^j$ represents the total wage income, and TR_t is the dividends and transfers to households. Maximizing the utility function subject to a sequence of budget constraints, households make optimal consumption-saving and labor supply decisions. First-order conditions for consumption and bonds holding imply the following Euler equation:[5]

$$\varepsilon_t^c \left(C_t - \chi C_{t-1}\right)^{-\sigma_c} = \beta\left[\varepsilon_{t+1}^c \left(C_{t+1} - \chi C_t\right)^{-\sigma_c} R_t \frac{P_t}{P_{t+1}}\right] \tag{8}$$

Similarly for the foreign economy:

$$\varepsilon_t^{c*} \left(C_t^* - \chi C_{t-1}^*\right)^{-\sigma_c} = \beta\left[\varepsilon_{t+1}^{c*} \left(C_{t+1}^* - \chi C_t^*\right)^{-\sigma_c} R_t^* \frac{P_t^*}{P_{t+1}^*}\right] \tag{9}$$

The complete-market assumption implies that the marginal rate of substitution between consumption in the two countries is equalized:

$$\frac{\varepsilon_{t+1}^{c*} U_C\left(C_{t+1}^*\right)}{\varepsilon_t^{c*} U_C\left(C_t^*\right)} \frac{P_t^*}{P_{t+1}^*} \frac{S_t}{S_{t+1}} = \frac{\varepsilon_{t+1}^c U_C\left(C_{t+1}\right)}{\varepsilon_t^c U_C\left(C_t\right)} \frac{P_t}{P_{t+1}} \tag{10}$$

The equation presented above illustrates the equality of nominal wealth in both countries in all states and time periods. Because domestic and foreign agents are identical *ex ante* so that agents' marginal utility of income are equal, the international risk-sharing condition can be also written as: $\frac{\varepsilon_t^{c*} U_C(C_t^*)}{\varepsilon_t^c U_C(C_t)} = k \frac{S_t P_t^*}{P_t}$, where the real exchange rate is defined as $\mathrm{RS}_t = \frac{S_t P_t^*}{P_t}$ (where S_t is the nominal exchange rate defined as a unit of foreign currency in terms of the domestic one) and k is a constant that depends on initial conditions $\left(k \equiv U_C\left(C_0^*\right) P_0 / U_C(C_0) P_0^* S_0\right)$. In a model with flexible exchange rate regime, the risk-sharing equation determines the endogenous path of the exchange rate. In the monetary union specification (when nominal exchange rate is fixed) this equation can be viewed as a condition restricting the long-run divergence of consumption across borders. In particular, in the two-country setting when economies have a comparable size, this equation (together with the domestic Euler equation) can be used to pin down foreign consumption. However, in the small-economy framework, foreign consumption should be exogenous from the home economy perspective. Thus the separate Euler equation for the foreign country or the exogenous process for consumption (output) should be used. In addition, note that completeness of financial markets in the currency union implies the equality of the nominal interest rates across countries at all times, i.e. $R_t = R_t^*, \forall t$.

[5] Dropping the j index.

Firms: marginal cost and pricing decisions

Each firm, which is a monopolistic producer of a differentiated good, uses the following technology:

$$Y_{h,t}(z) = A_t L_t(z)^{1-\lambda} \tag{11}$$

where $L_t(z)$ is a composite labor input measured by hours worked; A_t is total factor productivity with $\varepsilon_t^a \equiv \log(A_t)$ and $\varepsilon_t^a = \rho\varepsilon_{t-1}^a + \upsilon_t$, where υ_t is i.i.d. shock with zero mean.

The solution to the profit maximization problem enables expressing the real marginal cost (in terms of domestic prices) in the following way:

$$\mathrm{MC}_t^r = \frac{\mathrm{MC}_t}{P_{H,t}} = (1-\lambda)^{-1}(A_t)^{-1} W_t^r \frac{P_t}{P_{H,t}} L_t(z)^{\lambda} \tag{12}$$

where $W_t^r = W_t/P_t$ denotes the real wage.

The domestic firm sets the price $p_h(z)$ and takes as given P, P_H, P_F and C. The price-setting behavior is modeled according to Calvo (1983). This type of pricing scheme is widely used in the current generation of DSGE models.[6] Each time period a fraction $\gamma^p \in [0,1)$ of randomly picked producers in country H are not allowed to change their prices. Thus the parameter γ^p reflects the level of price stickiness. The remaining fraction $(1-\gamma^p)$ can choose the optimal sector-specific price by maximizing the expected discounted value of profits subject to the demand function derived from the expenditure minimization problem. The optimal price, $\tilde{p}_{h,t}(z)$, is derived from the first-order conditions, which take the following form:

$$E_t \sum_{i=0}^{\infty} (\gamma^p \beta)^i \Lambda_{t,i} \left(\frac{p_h(z)}{P_H}\right)^{-\sigma} Y_H \left[\mathrm{MC}_{t+i}^r - \frac{1}{\mu_i^p}\frac{\tilde{p}_{h,t}(z)}{P_{H,t}}\right] = 0 \tag{13}$$

where $\mu_i^p = \frac{\sigma}{(1-\tau_i)(\sigma-1)}$ represents the overall degree of monopolistic distortion and leads to a wedge between price and the marginal costs. Benigno and Benigno (2003) and De Paoli (2009) refer to this gap as the markup shock, which fluctuates due to time variation of the tax rate. A Calvo-type setting implies the following law of motion for the price indices:

$$P_{H,t} = \left[\gamma^p (P_{H,t-1})^{1-\sigma} + (1-\gamma^p)\, \tilde{p}_{h,t,}(z)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{14}$$

Similar conditions can be derived for the producers in country F.

Labor decisions and wage setting

Firm z chooses a sequence of different types of labor h to minimize the total cost of production subject to the production technology (11). The solution to cost minimization implies the following equation for the demand for labor:

$$l_t(h,z) = \frac{1}{n}\left(\frac{W_t(h)}{W_t}\right)^{-\sigma_{w,t}} L_t(z) \tag{15}$$

where W_t is the aggregate wage index (minimizing expenditures needed to purchase one unit of labor L_t).

Following Erceg *et al.* (2000), we introduce staggered wage contracts into the model. In particular, each period the wage rate of a given type h can be reset optimally with the probability $1-\ \gamma^w$. The fraction γ^w of wage rates that cannot be optimized is set equal to the previous period wages, i.e. $W_t(h) = W_{t-1}(h)$. Thus the parameter γ^w represents the measure of the nominal wage rigidities. The optimal choice of wage $\tilde{W}_t(h)$ brings about a maximization of the expected household utility (1) subject to the sequence of budget constraints (7) and a sequence of demand schedules of the form (15). The first-order conditions can be written as

$$E_t \sum_{i=0}^{\infty} (\gamma^w \beta)^i \left\{\left(\frac{l_{t+i,t}(h)}{(C_{t+i} - \chi C_{t+i-1})^{-\rho}}\right)\left(\frac{\tilde{W}_{t+i,t}(h)}{P_{t+i}} - \mu_{w,t+i}^n \mathrm{MRS}_{t+i,t}\right)\right\} = 0 \tag{16}$$

where $l_{t+i,t}(h)$ denotes period $t+i$ labor inputs of workers whose wage was last reoptimized in period t; $\mathrm{MRS}_t = -\frac{U_{L,t}}{U_{C,t}} = \varepsilon_t^l (C_t - \chi C_{t-1})^{\sigma_C} l_t(h)^{\eta}$ is the marginal rate of substitution between consumption and labor. Finally, $\mu_{w,t+i}^n \equiv \frac{\sigma_{w,t}}{(\sigma_{w,t}-1)}$ is the natural (or desired) wage markup that would prevail under the flexible wages assumption. Time variation of this parameter leads to changes in workers' market power. The solution $\tilde{W}_t(h)$ will be the same for all wage-optimizing agents. Thus the index h can be dropped.

[6] Smets and Wouters (2003); Gali and Monacelli (2005).

Similarly to the price equation, the aggregate wage index can be written as follows:

$$W_t = \left[\gamma^w \left(W_{t-1}\right)^{1-\sigma_{w,t}} + \left(1-\gamma^w\right) \tilde{W}_t(h)^{1-\sigma_{w,t}}\right]^{\frac{1}{1-\sigma_{w,t}}} \tag{17}$$

Unemployment dynamics
Unemployment is introduced into the model following the approach presented in recent papers by Gali (2011a, 2011b) and GSW. Consider a household j who supplies labor of type h. The condition that determines the participation of the individual in the labor market can be obtained using the welfare optimization criteria (and taking as given wages set on the labor market). More specifically, the household will work only if his marginal utility of consumption (per unit of value) will be greater or equal to his marginal disutility of work, i.e.

$$\frac{\left(C_t^j - \chi C_{t-1}\right)^{-\sigma_c}}{P_t} \geq \frac{\varepsilon_t^l l_t(h)^\eta}{W_t(h)}$$

In a symmetric equilibrium the supply of type h labor $l^S(h)$ will be determined by a standard intratemporal optimality condition:

$$\frac{W_t(h)}{P_t} = \varepsilon_t^l \left(C_t - \chi C_{t-1}\right)^{\sigma_c} l^S(h)^\eta \tag{18}$$

Aggregating over labor types, we can interpret $\tilde{L}^S$ as the measure of the potential labor force (maximum level of labor employment rate). Then the aggregate unemployment rate at period t is defined as the log difference between the labor force and the actual labor employed:

$$u_t \equiv \ln\left(\tilde{L}_t^S\right) - \ln(L_t) \tag{19}$$

Such a definition of the unemployment rate is taken for practical purposes and, given the low observed unemployment rates, is very close to the conventional level given by $1 - L_t/\tilde{L}_t^S$.[7] The formulation of unemployment presented here is linked to the concept of involuntary unemployment. In particular, unemployed workers include all the individuals who would like to participate in the labor market (given the current conditions) but are not currently employed. [8]

We would like to note some differences between the modeling approach presented here and the specification of GSW. In particular, the latter is written in terms of employment rather than hours worked. A reformulation of the model with the different measure of the labor input introduces certain changes in the presentation of consumer preferences but does not affect the functional form of resulting model equations. We did estimate the model totally formulated in terms of employment, thus exactly replicating the setup of GSW. However, in our case, using hours as the labor input and introducing the equation linking hours and employees improves the fit of the model. At the same time, our model (implicitly) contains a simplifying assumption that employed and unemployed individuals want to work the same amount of hours. For this reason, equation (19) can be equivalently written in terms of employment as in GSW.

Real exchange rate decomposition and PPP violation
The real exchange rate in the model of a currency union is defined as a relative price of foreign and home goods and is equal to $\mathrm{RS}_t = P_t^*/P_t$. We assume that the law of one price holds for differentiated goods, i.e. $p_h(z) = p_h^*(z)$ and $p_f(z) = p_f^*(z)$. This in turn implies that $P_H = P_H^*$ and $P_F = P_F^*$. However, our model specification implies violation of the purchasing power parity (PPP) at the aggregate price level, i.e. $P \neq P^*$ and thus $\mathrm{RS} \neq 1$. We use the price indexes to express the real exchange rate as a function of relative prices and preference parameters. The real exchange rate can then be presented as

[7] For unemployment rates near zero, the following approximation applies: $1 - L_t/\tilde{L}_t^S = 1 - \exp\{-u_t\} \simeq u_t$.
[8] GSW admit that, in their model, unemployed individuals will receive a higher utility *ex post*, since their consumption will be the same and, in addition, they will not experience a disutility from work. Such a result is an unavoidable consequence of the assumption of full consumption risk sharing among individuals, which was made in order to preserve the representative household framework and ensure tractability.

$$\mathrm{RS} = \left(\frac{\upsilon^* + (1-\upsilon^*)\,(P_{FH})^{1-\theta}}{\upsilon + (1-\upsilon)(P_{FH})^{1-\theta}} \right)^{\frac{1}{1-\theta}} \tag{20}$$

where $P_{FH} = \frac{P_F}{P_H}$ denotes the terms of trade. Such a decomposition enables analysis of the source of the PPP violation. In particular, under $\upsilon \neq \upsilon^*$, the RS is affected by the terms of trade. For the small open-economy model specification, given the assumptions on υ and υ^*, the difference in country sizes necessarily results in different shares of consumption of home-produced goods in countries H and F. This so-called home bias channel of the PPP violation has also been previously analyzed by De Paoli (2009) and Sutherland (2002). The violation of PPP implies that fluctuations in the real exchange rate may result in a divergence in consumption across countries even under optimal risk sharing.

Market clearing and aggregate demand

The condition for goods market clearing in the small open economy is given by

$$Y_t(z) = \int_{j=0}^{n} c_h(z)\mathrm{d}j + \int_{j^*=n}^{1} c_h^*(z)\mathrm{d}j^* \tag{21}$$

where $c_h(z)$ and $c_h^*(z)$ represent individual domestic and foreign demand for good $z \in (0, n]$ produced at the home economy. The total demand in the rest of the world (country F) is given in a similar fashion. The total demand is obtained by substituting the corresponding demand functions (4) and (5) in (21). In order to obtain the small open-economy version of the general two-country framework, we apply the assumptions $\upsilon^* = n \cdot \alpha$ and $(1-\upsilon) = (1-n) \cdot \alpha$ and take the limit $n \to 0$ similar to De Paoli (2009). The resulting demand equations are given by the following expressions:

$$Y_t = \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} \times \left\{ (1-\alpha) C_t + \left(\frac{1}{\mathrm{RS}_t} \right)^{-\theta} \alpha C_t^* \right\} + G_{H,t} \tag{22}$$

$$Y_t^* = \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} \times C_t^* + G_{F,t}^* \tag{23}$$

where G and G^* are country-specific exogenous demand (government spending) shocks.The demand equations presented above illustrate the small open-economy implications. In particular, the demand for goods produced at home depends on both domestic and foreign consumption as well as the relative prices, whereas the demand for foreign-produced goods is not affected by changes in home consumption.

Government policy

We assume that exogenous demand (government spending) in the domestic economy follows a first-order autoregressive process with i.i.d. normal error term and (as in Smets and Wouters, 2007) is also affected by the productivity shock:

$$\widehat{g}_t^h = \rho_g \widehat{g}_{t-1}^h + \rho_{ga} \widehat{\varepsilon}_t^a + \epsilon_t^g \tag{24}$$

where $\widehat{g}_t^h \equiv \log(G_{H,t})$. The assumption $\rho_{ga} > 0$ is empirically motivated by the fact that government spending may include components affected by domestic productivity developments.

Since the small open economy is assumed to belong to the common currency area, the local authority does not conduct an independent monetary policy. Thus the interest rate is common for domestic and foreign economies. It is set by the union-wide monetary authority following the Taylor rule,[9] based on the economic performance of the whole EMU. More specifically, the interest rate is gradually adjusted in response to the deviations of area-wide CPI inflation and demand (current and past dynamics) from their steady-state levels:

$$\widehat{R}_t^* = \omega_r \widehat{R}_{t-1}^* + (1-\omega_r)\left(\psi_\pi \pi_t^* + \psi_y \widehat{y}_t^* + \psi_{\Delta y}\left(\widehat{y}_t^* - \widehat{y}_{t-1}^* \right) \right) + \widehat{\varepsilon}_t^r \tag{25}$$

and

$$\widehat{R}_t = \widehat{R}_t^* \tag{26}$$

where $\widehat{R}_t \equiv \log(R_t)$, ω_r is the interest rate smoothing parameter and $\widehat{\varepsilon}_t^r$ is the interest rate shock which follows an AR(1) process with ϵ_t^r i.i.d. normal error term.

[9] The specification of the policy rule (25) is standard and widely used in the modern DSGE literature (Smets and Wouters, 2003, 2007).

EQUILIBRIUM CONDITIONS: LOG-LINEAR REPRESENTATION

The scale of the model suggests that it does not have a closed-form solution. Hence we rely on linearization to obtain an approximate solution. The procedure consists of computing a first-order approximation of the model around its non-stochastic steady state. In this section, we present a log-linearized version of the main structural equations. We define $\widehat{x_t} \equiv \ln \frac{X_t}{X}$ as the log deviation of the equilibrium variable X_t under sticky prices and wages from its steady-state value. Moreover, we define the price and wage changes as $\Pi_H = \frac{P_{H,t}}{P_{H,t-1}}$ and $\Pi_W = \frac{W_t}{W_{t-1}}$; consequently, the producer price and wage inflation rates are $\pi_{H,t} \equiv \ln\left(\frac{P_{H,t}}{P_{H,t-1}}\right)$ and $\pi_{W,t} \equiv \ln\left(\frac{W_t}{W_{t-1}}\right)$. We approximate the model around the steady state, in which $\overline{G} = 0$, $\mu^p \geq 1$ and producer prices and wages do not change, i.e. $\Pi_H = 1$ and $\Pi_W = 1$ at all times. In addition, $\overline{RS} = 1$, $\overline{C} = \overline{C}^*$, $\overline{Y} = \overline{Y}^*$.

The dynamics of consumption follow from the consumption Euler equation (8) and in the log-linearized form is given by

$$\widehat{c}_t = \frac{1}{(1+\chi)} E_t\left[\widehat{c}_{t+1}\right] + \frac{\chi}{(1+\chi)}\widehat{c}_{t-1} - \frac{(1-\chi)}{\sigma_c(1+\chi)}\left(\widehat{R}_t - E_t\left[\widehat{\pi}_{t+1}\right] + \widehat{\widehat{\varepsilon}}{}_t^c\right) \tag{27}$$

where $\widehat{\widehat{\varepsilon}}{}_t^c = \frac{(1-\chi)}{\sigma_c(1+\chi)}\left(\widehat{\varepsilon}_t^c - \widehat{\varepsilon}_{t+1}^c\right)$. The backward-looking term arises in the consumption equation due to the assumption of external habit formation captured by the parameter χ. Therefore, current consumption $(\widehat{c}_t)$ depends on a weighted average of past and expected future consumption. The consumption process is also affected by the *ex ante* real interest rate $\left(\widehat{R}_t - E_t\left[\widehat{\pi}_{t+1}\right]\right)$ and a disturbance term $\widehat{\widehat{\varepsilon}}{}_t^c$, which is assumed to follow a first-order autoregressive process with an i.i.d.-Normal error term: $\widehat{\widehat{\varepsilon}}{}_t^c = \rho_c \widehat{\widehat{\varepsilon}}{}_{t-1}^c + \epsilon_t^c + \rho_{cf}\epsilon_t^{c*}$. We also assume that the domestic shock is affected by the foreign consumption disturbance.[10]

The optimal price-setting condition (13) combined with equation (14) gives rise to the following New-Keynesian Phillips curve, which describes the dynamics of the domestic inflation in terms of the real marginal costs:

$$\widehat{\pi}_{H,t} = \beta E_t\left[\widehat{\pi}_{H,t+1}\right] + \frac{(1-\gamma^p\beta)(1-\gamma^p)}{\gamma^p}\left(\widehat{\mathrm{mc}}_t^r\right) + \widehat{\mu}_{p,t} \tag{28}$$

The price markup disturbance $\left(\widehat{\mu}_{p,t}\right)$ is assumed to follow an AR(1) process: $\widehat{\mu}_{p,t} = \rho_p \widehat{\mu}_{p,t-1} + \epsilon_t^p$, where ϵ_t^p is an i.i.d.-normal price markup shock. The marginal cost is obtained by log-linearizing equation (12) and is given by

$$\widehat{\mathrm{mc}}_t^r = \widehat{w}_t^r + \lambda \widehat{L}_t - \widehat{p}_{H,t} - \widehat{\varepsilon}_t^a \tag{29}$$

where $p_{H,t} = P_{H,t}/P_t$ denotes domestic relative price. The characterization of real marginal costs in the open economy setting is somewhat different from that of the closed economy due to the impact of relative prices, which reflect the distinction between domestic and consumer prices. Note that even though the forward-looking specification of inflation (28) does not imply any intrinsic inertia, the evolution of the marginal cost is affected by the price inertia,[11] which comes from the Calvo-type process of price setting.

Log-linearizing the optimal wage-setting condition (16) and the law of motion for the wage rate (17) allows us to obtain the following equation for wage inflation:

$$\widehat{\pi}_t^W = \beta E_t\left[\widehat{\pi}_{t+1}^W\right] - \frac{(1-\gamma^w\beta)(1-\gamma^w)}{\gamma^w(1+\sigma_w\eta)}\left(\widehat{\mu}_{w,t} - \widehat{\mu}_{w,t}^n\right) \tag{30}$$

where $\widehat{\mu}_{w,t}^n$ is the desired wage markup:

$$\widehat{\mu}_{w,t} = \widehat{w}_t^r - \widehat{\mathrm{mrs}}_t \tag{31}$$

and $\widehat{\mathrm{mrs}}_t = \widehat{\varepsilon}_t^l + \frac{\sigma_C}{1-\chi}\left(\widehat{c}_t - \chi\widehat{c}_{t-1}\right) + \eta\widehat{L}_t$. The wage markup disturbance $\widehat{\mu}_{w,t}^n$ is assumed to follow an i.i.d.-Normal process: $\widehat{\mu}_{w,t}^n = \widehat{\epsilon}_t^w$. Using the definition of the wage inflation $\widehat{\pi}_t^W = \widehat{w}_t - \widehat{w}_{t-1}$, we can write down the expression for the dynamics of the real wages as follows:

$$\widehat{w}_t^r = \frac{1}{(1+\beta)}\left\{\begin{array}{c}\widehat{w}_{t-1}^r + \beta E_t\left[\widehat{w}_{t+1}^r\right] - \widehat{\pi}_t + \beta E_t\left[\widehat{\pi}_{t+1}\right] \\ +\frac{(1-\gamma^w\beta)(1-\gamma^w)}{\gamma^w(1+\sigma_w\eta)}\left[\frac{\sigma_C}{1-\chi}\left(\widehat{c}_t - \chi\widehat{c}_{t-1}\right) + \eta\widehat{L}_t + \widehat{\varepsilon}_t^l - \widehat{w}_t^r\right]\end{array}\right\} + \widehat{\mu}_{w,t}^n \tag{32}$$

[10] In such a way we introduce 'one-way' correlation between domestic and foreign consumption shocks. Such an assumption, however, is not crucial for the estimation and forecasting results.
[11] The degree of inertia depends on the value of the Calvo parameter in equation (14).

where $\widehat{\varepsilon}_t^l = \log\left(\varepsilon_t^l\right)$ is labor supply shock. which is assumed to follow an ARMA(1,1) process: $\widehat{\varepsilon}_t^l = \rho_l \widehat{\varepsilon}_{t-1}^l - \rho_{ma,l}\epsilon_{l,t-1} + \epsilon_t^l$.

Equation (30) demonstrates that the evolution of wage inflation is determined by fluctuations of the wedge between the actual and desired wage markups. In particular, when the markup charged is higher than the natural level, wages will respond negatively. The dynamics of the markup is driven by fluctuations in the real wage and the marginal rate of substitution. In particular, due to the presence of nominal wage stickiness, real wages adjust only gradually to the desired wage markup. In addition, equation (32) shows that the real wage dynamics are affected by CPI inflation. An increase in the inflation rate will result in a decline of real wages and a contraction in the wage markup. As a consequence, higher expected inflation rate (translated into lower expected wage markup) will motivate workers to set higher nominal wages today to offset the possible reduction of the real wages in the future.

In order to describe the unemployment dynamics, we log-linearize equations (18) and (19) and obtain the following expressions:[12]

$$\widehat{w}_t^r = \widehat{\varepsilon}_t^l + \frac{\sigma_C}{1-\chi}\left(\widehat{c}_t - \chi\widehat{c}_{t-1}\right) + \eta\widehat{\tilde{L}_t^S} \tag{33}$$

and

$$\widehat{u}_t = \widehat{\tilde{L}_t^S} - \widehat{L}_t \tag{34}$$

A common problem with European data is the absence of consistent data on aggregate hours. Therefore, following a number of studies performed for the euro area, we use employment instead of 'hours worked' in the estimation procedure. The employment time series is normally more persistent compared to hours. Thus, following Smets and Wouters (2003), we assume hours to be flexible whereas rigidity in employment gives rise to the following Calvo-type auxiliary equation which links these two measures of labor input:

$$\widehat{\mathrm{Em}}_t = \beta\widehat{\mathrm{Em}}_{t+1} + \frac{(1-\gamma^m\beta)(1-\gamma^m)}{\gamma^m}\left(\widehat{L}_t - \widehat{\mathrm{Em}}_t\right) + \widehat{\varepsilon}_t^{\mathrm{em}} \tag{35}$$

where $\widehat{\mathrm{Em}}_t$ denotes the number of people employed and γ^m denotes the fraction of firms that can adjust employment to the desired level. $\widehat{\varepsilon}_t^{\mathrm{em}}$ is an exogenous shock to the employment, which follows an AR(1) process.

The demand for labor is represented by the following expression, based on the first-order approximation of the condition (11):

$$(1-\lambda)\widehat{L}_t = \widehat{Y}_t - \widehat{\varepsilon}_t^a \tag{36}$$

The log-linear representation of equation (22) describes the aggregate demand for domestic goods:

$$\widehat{Y}_t = -\theta\widehat{p}_{H,t} + (1-\alpha)\widehat{c}_t + \alpha\widehat{c}_t^* + \theta\alpha\widehat{\mathrm{RS}}_t + \widehat{g}_t^h \tag{37}$$

where $\widehat{g}_t^h$ is given by equation (24).

The real exchange rate is given by the following expression:

$$\widehat{\mathrm{RS}}_t = (1-\alpha)\widehat{p}_{FH,t} \tag{38}$$

where $\widehat{p}_{FH,t}$ denotes the terms of trade. Moreover, from the price index relation it follows that

$$\widehat{p}_{H,t} = -\alpha\widehat{p}_{FH,t} \tag{39}$$

The relationship between CPI, domestic inflation and the terms of trade is given by

$$\pi_t = \pi_{H,t} + \alpha\Delta\widehat{p}_{FH,t} \tag{40}$$

[12] It is easy to show that there exists the following relationship between wage markup and unemployment rate: $\widehat{\mu}_{w,t} = \eta\widehat{u}_t$. Therefore, the wage inflation equation can be reformulated in terms of the unemployment rate, which can enter the set of observable variables. As GSW point out, such a representation allows an important identification problem to be overcome, which limits the use of New Keynesian models for policy analysis. In particular, without an explicit measure of unemployment (or alternatively labor supply), the wage markup disturbance and the preference shock that affects the labor disutility cannot be distinguished. Such an identification problem may result in inaccurate policy recommendations, because these shocks call for qualitatively different optimal policy responses.

Finally, the evolution of the real exchange rate takes the form

$$\widehat{\mathrm{RS}}_t - \widehat{\mathrm{RS}}_{t-1} = \pi_t^* - \pi_t + \widehat{\varepsilon}_t^{rs} \tag{41}$$

where π_t^* is CPI inflation in the foreign country[13] and $\widehat{\varepsilon}_t^{rs}$ is an exogenous shock, which captures the developments in other types of relative prices at home and abroad that affect the evolution of the real exchange rate and consumer price inflation but not modeled here explicitly.[14] $\widehat{\varepsilon}_t^{rs}$ is assumed to follow a first-order autoregressive process with an i.i.d.-Normal error term: $\widehat{\varepsilon}_t^{rs} = \rho_{rs}\widehat{\varepsilon}_{t-1}^{rs} + \epsilon_t^{rs}$.

In this version of the paper we consider a simplified (three-equation) structure for the foreign economy, associated with the euro area. Also, we do not focus on asymmetries between the domestic economy and the rest of the world. Thus we assume the same values of such parameters as habit formation and preferences for home and foreign economies. Calvo price rigidities and exogenous processes are country specific. Foreign inflation is governed by the following simplified Phillips curve relation:

$$\widehat{\pi}_t^* = \beta E_t\left[\widehat{\pi}_{t+1}^*\right] + \frac{(1-\gamma^{p*}\beta)(1-\gamma^{p*})}{\gamma^{p*}}\left(\sigma_C\widehat{c}_t^* + \eta\widehat{y}_t^*\right) + \widehat{\varepsilon}_t^{\pi*} \tag{42}$$

The dynamics of foreign consumption is derived from log-linearization of equation (8a):

$$\widehat{c}_t^* = \frac{1}{(1+\chi)}E_t\left[\widehat{c}_{t+1}^*\right] + \frac{\chi}{(1+\chi)}\widehat{c}_{t-1}^* - \frac{(1-\chi)}{\sigma_c(1+\chi)}\left(\widehat{R}_t^* - E_t[\widehat{\pi}_{t+1}^*] + \widehat{\widehat{\varepsilon}}_t^{C*}\right) \tag{43}$$

where $\widehat{\widehat{\varepsilon}}_t^{C*}$ denotes foreign preference consumption shock, which is assumed to follow an AR(1) process: $\widehat{\widehat{\varepsilon}}_t^{C*} = \rho_{c*}\widehat{\widehat{\varepsilon}}_{t-1}^{C*} + \epsilon_t^{c*}$. Foreign demand is obtained by log-linearization of equation (27):

$$\widehat{y}_t^* = \widehat{c}_t^* + \widehat{g}_t^* \tag{44}$$

Finally, the nominal interest rate dynamics are given by equation (29). Note that foreign dynamics are completely exogenous from the small open-economy perspective. In the estimation procedure we include only three time series related to the foreign economy (inflation, output and interest rate). Therefore, certain shocks can be poorly identified. For this reason, we assume no foreign government spending shock, $\widehat{g}_t^* = 0$. Moreover, foreign productivity and price markup shocks are not identified separately. Thus we consider their aggregated impact on foreign inflation.

The complete set of linearized equilibrium conditions is given by:

- equations (26)–(41), which describe the evolution of 16 domestic endogenous variables;
- equations (25), (42), (43) and (44), which describe the evolution of the foreign variables;
- equations that describe the evolution of 11 exogenous shocks:
  $\widehat{\varepsilon}_t^c$, $\widehat{\varepsilon}_t^a$, $\widehat{\mu}_{p,t}$, $\widehat{\mu}_{w,t}^n$, $\widehat{\varepsilon}_t^l$, $\widehat{\varepsilon}_t^{em}$, $\widehat{g}_t^h$, $\widehat{\varepsilon}_t^{rs}$, $\widehat{\varepsilon}_t^r$, $\widehat{\widehat{\varepsilon}}_t^{C*}$, $\widehat{\varepsilon}_t^{\pi*}$.

ESTIMATION STRATEGY AND RESULTS

Data

We use quarterly time series for Luxembourg for the following macro-economic variables: real GDP, employment (residents and non-residents employed by resident producer units), compensation per employee (working in a resident production unit), consumer price index, unemployment rate and real effective exchange rate (REER; CPI deflated). The first two variables are expressed in per capita terms.[15] The foreign variables are real GDP, euro area short-term nominal interest rate and CPI inflation. All variables (except the nominal interest rate) are seasonally adjusted and log differenced. The sample is from 1995:Q1 to 2011:Q3 since quarterly data are not available before 1995. The time series of real wages is constructed as compensation per employee divided by consumer prices. All variables have been demeaned prior to estimation.

[13] In the small open-economy specification presented here, $\pi^* = \pi^F$.
[14] For example, relative price of non-tradable goods.
[15] In Luxembourg, the structure of the labor market is very specific. In particular, the fraction of non-residents is significant and constitutes about 40% of the salaried employment. Therefore, variables presented in 'per capita' (i.e. 'per resident person') terms do not properly account for the complexity of the employment structure and thus represent rather an approximation of the regular per capita variables. At the same time, we believe that such a generalization does not significantly affect the results of our estimation because, by detrending, we remove the effect of (possibly) different trends in the evolution of residents and non-residents and focus on the analysis of the cyclical component (which is assumed to have similar dynamic properties for both groups of employees).

Data on the real exchange rate are taken from the IMF International Financial Statistics. The source for unemployment rate is the OECD Statistics. The rest of the data are taken from STATEC national accounts.

Using the dataset described above, we estimate and compare the forecasting performance for the following model specifications:

- DSGE model presented above ('Equilibrium conditions: log-Linear representation');
- unrestricted VAR;
- univariate AR(2);
- Bayesian VAR(2);
- DSGE-VAR(2) model.

The rest of the section is organized as follows. Following is a general description of the DSGE model estimation procedure as well as the results. We then briefly describe the alternative (to DSGE) forecasting models mentioned above and compare the fit of the DSGE and DSGE-VAR models.

DSGE model: estimation results

In this subsection we describe the estimation results of the DSGE structural model presented in the previous section. The model is estimated using Bayesian techniques (see, for example, Schorfheide, 2011). We use Dynare version 4.3 to estimate the model. For estimation purposes, the log-linearized DSGE model presented in the previous section is augmented by the following measurement equations, which relate the model variables to the vector of observables:

$$\begin{bmatrix} \Delta \ln \mathrm{RGdp}_t \\ \Delta \ln P_t \\ \Delta \ln \mathrm{REER}_t \\ \Delta \ln \mathrm{RWage}_t \\ \Delta \ln \mathrm{Empl}_t \\ \Delta \ln \mathrm{Unempl}_t \\ \mathrm{STN}_t \\ \Delta \ln P_t^* \\ \Delta \ln \mathrm{RGDP}_t^* \end{bmatrix} = \begin{bmatrix} \widehat{y}_t - \widehat{y}_{t-1} \\ \widehat{\pi}_t \\ \widehat{\mathrm{RS}}_t - \widehat{\mathrm{RS}}_{t-1} \\ \widehat{w}_t^r - \widehat{w}_{t-1}^r \\ \widehat{\mathrm{Em}}_t - \widehat{\mathrm{Em}}_{t-1} \\ \widehat{u}_t - \widehat{u}_{t-1} \\ \widehat{R}_t^* \\ \widehat{\pi}_t^* \\ \widehat{y}_t^* - \widehat{y}_{t-1}^* \end{bmatrix} \tag{45}$$

On a theoretical level, the Bayesian approach to estimation takes the observed data as given, and treats the parameters of the model as random variables. In general terms, the estimation procedure involves solving the linear rational expectations model described above. The solution can be written in a state space form, which consists of the system of state equations augmented by the observation (measurement) equations (45). The likelihood of the linearized DSGE model is built up by generating forecasts from the state-space system with the use of the Kalman filter. In particular, the Kalman filter generates projections or forecasts of the states of the linear approximate solution of the DSGE model given an information set of observed macro time series.[16] Forecasts of these observables are also produced by the Kalman filter. The Kalman filter is useful for evaluating the likelihood of a linearized DSGE model because the forecasts are optimal within the class of all linear models (when shock innovations are assumed to be normally distributed). Posterior distributions of the structural parameters are formed by combining the likelihood function of the data with prior densities, which contain information about the model parameters obtained from other sources (microeconometric, calibration and cross-country evidence), thus allowing extension of the relevant data beyond the time series that are used as observables. An additional benefit of using prior information is that it allows the steering of parameter estimates towards values that are considered to be 'reasonable' by the literature and to regularize highly nonlinear and often multi-modal posterior distributions. This second advantage is very important when comparing Bayesian methods to alternative estimation strategies such as maximum likelihood. Finally, numerical methods such as Markov chain Monte Carlo (MCMC) are used to characterize the posterior with respect to the model parameters. See Smets and Wouters (2003, 2007), Dynare Manual and An and Schorfheide (2005) for more details on Bayesian estimation of DGSE models.

Calibration and priors

Following the recent DSGE and New Open Macroeconomy literature, we calibrate a number of parameters. In particular, the discount factor β is fixed at 0.99, which implies an annual steady-state real interest rate of 4%. The elasticity of substitution across the differentiated types of labor σ_w is set to 6, which implies a steady-state wage markup of about 20%. The elasticity of substitution between foreign and home goods θ is assumed to be unitary. The

[16] For more details on the state-space models and forecasting with the use of the Kalman filter, see Hamilton (1994).

Table I(a). Prior and posterior distribution of structural parameters for the baseline DSGE model

Parameters		Prior distribution			Posterior distribution				
		Type	Mean	SD	Mode	SD	Mean	5%	95%
Production function	λ	Beta	0.3	0.1	0.202	0.077	0.215	0.096	0.332
Degree of openness	α	Beta	0.3	0.15	0.102	0.034	0.106	0.051	0.161
Consumption utility	σ_c	Norm	1	0.375	1.256	0.292	1.283	0.816	1.75
Labor utility	η	Norm	2	1.5	2.873	0.804	3.45	2.065	4.883
Consumption habit	χ	Beta	0.5	0.15	0.776	0.062	0.777	0.677	0.875
Calvo prices	γ^p	Beta	0.75	0.15	0.923	0.022	0.919	0.884	0.957
Calvo wages	γ^w	Beta	0.75	0.15	0.929	0.019	0.933	0.899	0.967
Calvo employment	γ^m	Beta	0.75	0.15	0.918	0.021	0.914	0.875	0.951
Calvo foreign prices	γ^{p*}	Beta	0.75	0.15	0.977	0.01	0.977	0.962	0.992
Policy rule: lagged interest rate	ω_r	Beta	0.5	0.2	0.973	0.010	0.97	0.958	0.985
Policy rule: output	ψ_y	Gamma	0.25	0.125	0.201	0.101	0.25	0.075	0.414
Policy rule: lagged output	$\psi_{\Delta y}$	Gamma	0.25	0.125	0.151	0.034	0.155	0.094	0.212
DSGE prior weight [a]	$\tilde{w}$	Unif	0	10	1.880	0.442			

[a] DSGE prior weight parameter is estimated in DSGE-VAR(2) model specification.

policy rule parameter which determines the interest rate response to inflation is set to 1.5. In addition, we fix the standard deviation of the exogenous demand (government spending) shock at 0.1 and the autoregressive coefficient of the productivity shock at 0.9. The latter two parameters have been calibrated because the government spending shock is not separately identified and the productivity shock is imprecisely estimated. In our case, the reason for a weak identification of these stochastic processes can be related to the short data sample that turns out to be not informative enough and fails to introduce 'sufficient' curvature in the likelihood function in certain directions. In addition, we have to use employment data rather than hours worked (since the latter is not available) and link these two measures of the labor input via equation (35). Such an ad hoc relation can also distort the estimated productivity process. The calibrated values for the shocks have been chosen to approximate the standard deviation of the output growth from 1995 to 2011. Parameter identification is an important problem facing current generation of DSGE models that feature complex structure and, as a consequence, highly nonlinear relationships between the structural and reduced-form parameters. Thus the mapping between the two might be unknown and only an approximation can be obtained. In practice, lack of identification is a complex issue that can be related to the model specification, dimensionality of the problem, assumptions regarding the shock processes as well as the sample size.[17]

In the choice of priors, we mainly follow the original papers by Smets and Wouters (2003, 2007) as well as GSW. The first two papers present a careful description of the estimation methodology as well as the justification for the choice of priors. The estimation procedure starts with the estimation of the mode of the posterior distribution by maximizing the log posterior function. Secondly, the Metropolis–Hastings algorithm was used to compute the posterior distribution and to evaluate the marginal likelihood of the model. 100,000 MCMC draws have been performed using three chains.

Parameter estimates

A visual diagnostic of the estimation results can be found in Figure A.I in the Appendix, where we plot prior versus posterior distributions. Most of the parameters are identified as their posterior is significantly different from prior. For the majority of the parameters, the variance of the posterior is lower compared to the prior distribution, indicating that data are quite informative. In the case of no identification for a particular parameter, the likelihood function would be flat in the corresponding direction and the posterior distribution would be prior driven. Figure A.I illustrates that a policy rule parameter which determines the impact of output changes suffers from lack of identification. All the marginal posterior distributions are unimodal, which is one of the criteria for assessment of MCMC convergence. Metropolis–Hastings convergence graphs (not presented here) indicate that convergence for all parameters is efficient and fast.

Tables I(a) and I(b) report the estimates of the DSGE model parameters. The tables show the mode, which maximizes the posterior distribution, along with the approximate standard deviation computed from the inverse Hessian at the posterior mode. Furthermore, the tables present posterior statistics from MCMC—posterior means and the 95% probability intervals of the model parameters. Our estimate of the utility function parameter σ_c implies the value of intertemporal elasticity of substitution is less than one. Such an estimate is generally in line with the calibration made

[17] Canova and Sala (2009) investigate identification issues in DSGE models and their consequences for parameter estimation. They point out that small samples exacerbate the consequences of identification problems for estimation and inference.

Table I(b). Prior and posterior distribution of shock processes for the baseline DSGE model

Parameter		Prior distribution			Posterior distribution				
		Type	Mean	SD	Mode	SD	Mean	5%	95%
Standard deviations									
Consumption preference	υ_c	Inverse gamma	0.1	2	0.037	0.01	0.05	0.027	0.071
Productivity	υ_a	Inverse gamma	0.1	2	1.296	0.306	1.389	0.887	1.885
Price markup	υ_p	Inverse gamma	0.1	2	0.212	0.038	0.223	0.155	0.284
Wage markup	υ_w	Inverse gamma	0.1	2	0.54	0.049	0.553	0.47	0.636
Relative price	υ_{rs}	Inverse gamma	0.1	2	0.985	0.088	1.01	0.855	1.155
Labor supply	υ_l	Inverse gamma	0.1	2	0.108	0.033	0.135	0.073	0.193
Exogenous employment	υ_{em}	Inverse gamma	0.1	2	0.142	0.042	0.16	0.087	0.231
Foreign demand	υ_{c*}	Inverse gamma	0.1	2	0.071	0.017	0.081	0.052	0.11
Foreign prices	υ_{p*}	Inverse gamma	0.1	2	0.463	0.042	0.475	0.403	0.546
Interest rate	υ_r	Inverse gamma	0.1	2	0.08	0.011	0.086	0.065	0.106
Persistence and correlation									
Consumption	ρ_c	Beta	0.5	0.2	0.909	0.024	0.886	0.836	0.939
Price markup	ρ_p	Beta	0.5	0.2	0.368	0.122	0.364	0.171	0.566
Relative price	ρ_{rs}	Beta	0.5	0.2	0.184	0.087	0.201	0.062	0.33
Labor supply: AR	ρ_l	Beta	0.5	0.2	0.85	0.055	0.826	0.733	0.924
Labor supply: MA	$\rho_{ma,l}$	Beta	0.5	0.1	0.631	0.079	0.63	0.501	0.763
Exogenous employment	ρ_{em}	Beta	0.5	0.2	0.635	0.134	0.587	0.362	0.817
Interest rate	ρ_r	Beta	0.5	0.2	0.438	0.101	0.444	0.283	0.61
Foreign demand	ρ_{c*}	Beta	0.5	0.2	0.789	0.068	0.759	0.652	0.873
Demand–productivity	ρ_{ag}	Norm	0.5	0.25	0.785	0.173	0.786	0.521	1.049
Consumption–foreign demand	ρ_{cf}	Norm	0.5	0.25	0.468	0.160	0.515	0.247	0.772

in the majority of the RBC literature, which sets an elasticity of substitution between 0.5 and 1. Another parameter that determines the impact of the interest rate changes on consumption is habit formation, which is estimated to be 0.77. Such a relatively high value implies initially lower but more persistent response of consumption following changes in the short-term interest rate or consumption preference shock. The posterior mean of the habit parameter is somewhat higher than the estimates obtained in Smets and Wouters (2003), who report a value of 0.55, but is close to numbers from other studies performed on European data. In particular, Pytlarczyk (2005) finds a habit persistence estimate 0.68 for Germany and 0.8 for the rest of the euro area. Jondeau and Sahic (2004) estimate the multi-country euro area model and report values of 0.73 for France and 0.84 for Italy. The inverse of the elasticity of labor supply has the posterior mean equal to 3.45 which implies that the response of labor supply to changes in the wage rate is relatively small. The estimate of this parameter is close to the value of 4.0 reported in Gali *et al.* (2011). Together with the calibrated steady-state wage markup, the estimated value of the inverse Frisch elasticity is consistent with the average unemployment rate of about 5.8%.[18]

The degree of openness parameter is estimated at about 10%, which is somewhat lower than could be expected for such an open economy. The reason for such a result is an extreme dynamics of the terms of trade series for Luxembourg. In particular, the actual data imply an excessively high degree of openness that cannot be reasonably fitted into a theoretical model framework. Calibrating this parameter at relatively high levels leads to greater implied volatility of other real variables compared to the actual dynamics and thus results in a deterioration of the model fit.

Structural rigidities parameters, which are found to play a crucial role in capturing the business cycle fluctuations, are well identified. The estimates of the Calvo parameters at 0.91 for prices and 0.93 for wages imply an average duration of contracts of two and half years. These values are higher compared to micro-evidence for some European countries like Germany and also greater than estimates obtained by Smets and Wouters (2003, 2007) for the euro area and the USA respectively or Adolfson *et al.* (2008) for Sweden. At the same time, Burriel *et al.* (2010) report a similar estimate for Calvo price parameter for the Spanish economy. One factor that could explain the high degree of the price stickiness is the assumption of i.i.d. price and wage markup shocks. Smets and Wouters (2007) assume ARMA structure for these stochastic processes. However, in our case such an assumption is not supported by the data and reduces the marginal likelihood of the model. The absence of such factors as sluggish capital adjustment, which affect the process driving marginal costs, can bias upward the estimate of Calvo price stickiness. In our estimation exercise, we also tried to evaluate indexation parameters, which measure the proportion of prices/wages that cannot adjust in the current period but instead are indexed to the lagged inflation rates. Price indexation parameter is estimated at the low value, which is in line with the European evidence, and does not significantly affect the model likelihood. The wage indexation parameter is not separately identified from the parameter measuring the slope of the wage Phillips curve. Thus we have decided to abstract from modeling the indexation process.

[18] A sample mean in the data sample we consider.

Overall, the data are quite informative about the persistence and volatility of exogenous disturbances. The preference and labor supply shocks appear to be the most persistent, with AR(1) coefficients of 0.89 and 0.83 respectively. In general, the level of persistence of stochastic processes is not very high. Such a result indicates that the model contains a sufficiently persistent endogenous propagation mechanism. Regarding the estimates of the volatility of shocks, various studies do not seem to reach a consensus. The values of the parameters of stochastic processes is highly model dependent. In addition, many authors normalize structural shocks, which reduces their volatility. Our results suggest that productivity, relative price and wage markup shocks have the highest estimated standard deviations. As in GSW adding unemployment as an observable variable allows us to separately identify labor supply and wage markup shocks, which appear to have quite different stochastic properties. Such a result will translate into the differentiated impact of these shocks on the forecast error variance of real variables when explaining business cycle fluctuations.

Finally, turning to the parameters of the Taylor rule, there is a high degree of interest rate smoothing which is generally supported by the literature.[19] The monetary policy appears to respond relatively strongly to changes in output, with the posterior mean of the corresponding coefficient being equal to 0.15. The estimates of the inflation and output level reaction coefficients are driven by a prior. This can be partially explained by the relatively short data sample, which implies a higher weight on the prior information. In addition, we assume a highly simplified model of the foreign economy. However, such a lack of identification does not affect the overall results. Finally, we would like to note that our estimation sample ends at 2011:Q3 and thus includes the recent financial crisis observations. Our estimates can be affected to some extent, therefore, by the unconventional measures implemented by the monetary authority but not captured in this modeling framework. As a robustness check, we compare the parameters of the model estimated on a sample that ends in 2007:Q4 and on the full sample. The Tables A.I and A.II in the Appendix demonstrate that the parameters, especially those that determine the model persistence, do not differ significantly and thus our results are not driven by specific dynamics caused by inclusion of financial crisis observations.

Alternative forecasting models: description and comparison

In addition to the DSGE model, we estimate and compare the forecasting performance of the following model specifications:

- *Unrestricted VAR*. The model can be written in the following general form:

$$Y_t = \Phi_x X_t + \Phi_1 Y_{t-1} + \ldots + \Phi_p Y_{t-p} + u_t,\ u_t \sim i.i.d.\,N(0;\Sigma_u) \tag{46}$$

  where $p = 2$ to allow for sufficient dynamics without exhausting degrees of freedom, due to the rather small sample available. The vector of endogenous variables is the same as in DSGE estimation, i.e. $Y_t = [\Delta\ln(\text{Real GDP}),\ \Delta\ln(\text{CPI}),\ \Delta\ln(\text{Real.Effect.Exch.Rate}),$ $\Delta\ln(\text{Real wages}),\ \Delta\ln(\text{Employment}),\ \Delta\ln(\text{Unemployment})]$. In order to make the models comparable, in VAR forecasting we impose the small open-economy restriction, which implies that foreign variables are considered as exogenous, i.e. the vector of exogenous variables is $X_t = [\text{Nomin.Inter.rate},\ \Delta\ln(\text{Foreign GDP}),$ $\Delta\ln(\text{Foreign CPI})]$.
- *Univariate AR(2)*. Such a specification implies that the matrices of parameters Φ and variance–covariance matrix Σ_u in the VAR specification are diagonal.

The solution of the linearized DSGE model generates a restricted (and possibly misspecified) moving average representation for the vector of observed data Y_t. The MA representation can be approximated by a constrained VAR with p-lags and coefficient restrictions given by nonlinear functions of the DSGE parameter vector ϑ:

$$Y_t = \Phi_x^*(\vartheta)X_t + \Phi_1^*(\vartheta)Y_{t-1} + \ldots + \Phi_p^*(\vartheta)Y_{t-p} + u_t \tag{47}$$

Because of this close relationship between structural and reduced-form models, unconstrained VARs are widely used in the literature as a benchmark for evaluating the empirical validity of cross-equation restrictions imposed by the DSGE structure. On the one hand, VAR represents a flexible and unrestricted framework. At the same time, coefficient estimates can be very imprecise and forecasts have large standard errors due to the large number of parameters and short time series. The current literature addresses this problem by the use of Bayesian estimation techniques. In this paper we consider two types of priors on VAR coefficients: one is non-theoretical and another one is based on the DSGE model. The corresponding model specifications are described below.

- *Bayesian VAR(2)*. The model combines the VAR Likelihood function with the prior information summarized by the prior density. This approach represents a flexible way to reduce the dimensionality of the parameter space, incorporate additional information and thus decrease the parameter uncertainty. As a result, the forecasting

[19] Estimates vary depending on the estimation sample.

Table II. Model comparison in terms of log data density (LDD)

Model specification	DSGE	DSGE-VAR(2)	
	LDD	LDD	DSGE weight
Baseline: medium-scale DSGE	−577.54	−597.69	1.880
Baseline: medium-scale DSGE w/o unemployment	−395.44	−404.68	1.868
Small-scale DSGE w/o labor market block	−279.29	−280.37	1.142

performance can be improved over the standard VAR methods. In this paper we choose Sims–Zha Normal-Wishart priors (described in Sims and Zha, 1998), which proved to be the best practice in recent empirical studies. This BVAR specification combines a Minnesota-style prior (see Litterman, 1984) with priors that take into account the degree of persistence in the variables. Since we work with stationary data, the original Sims and Zha prior is adapted by setting the prior mean on the first own lag to zero for all the variables. We assume the standard values of hyperparameters found to work well in most forecasting applications: 'overall tightness' and the 'decay' parameter, which determine the rate at which prior coefficients decline as lag increases, are set to 1. The AR(1) tightness is set to 0.5, and the 'sum of coefficient prior weight' is set to 0.1.

- *DSGE-VAR(2)*, a sort of Bayesian approach to VAR that uses DSGE model restrictions to construct a micro-founded prior about VAR parameters and thus may improve VAR estimates by incorporating extra information. Alternatively, this method can be viewed as a way to improve the empirical properties of the DSGE model by relaxing tight cross-equation restrictions that might be at odds with real data. The idea of the approach is to simulate data from the DSGE model, append simulated to actual data and estimate a VAR on the extended sample. The optimal proportion (can be estimated) of simulated to actual data measures the weight on DSGE restrictions.[20]

Comparing the fit of the DSGE and DSGE-VAR models

The fit of a model estimated using Bayesian methods can be ascertained using marginal data density, defined as

$$p\,(Y|\mathcal{M}) = \int \mathcal{L}\,(\vartheta|Y)\; p_0(\vartheta)\mathrm{d}\vartheta$$

where $\mathcal{L}\,(\vartheta|Y)$ is the likelihood function of the data Y given parameters of the model ϑ, and $p_0(\vartheta)$ is the prior density. In other words, the marginal data density are simply an integral over the posterior density, where posterior is understood as likelihood times prior. This measure allows a straightforward comparison of several models estimated on the same data with respect to a reference model. To evaluate a marginal density of the data we can use a Gaussian approximation of the posterior function (so-called Laplace approximation), which takes the following form:

$$\widehat{p}\,(Y|\mathcal{M}) = (2\pi)^{\frac{k}{2}}\,|\Sigma_{\vartheta^m}|^{1/2}\,\mathcal{L}\,(\vartheta^m|Y)\; p_0\,(\vartheta^m)$$

where ϑ^m is the posterior mode. This technique is computationally efficient since only numerically calculated posterior mode and covariance of the estimated parameters are required. Another option to compute the marginal density is to use information from the MCMC runs and is typically referred to as the Modified Harmonic mean estimator. The idea is to simulate the marginal density and to simply take the average of these simulated values. In our estimation exercise, the two measures of marginal density are very close, which indicates that the posterior function is close to being symmetric and does not possess features such as fat tails, and therefore can be reasonably approximated by a multivariate normal distribution. Table II reports logarithms of marginal data densities for several DSGE model specifications that we have estimated. In particular, we estimate a baseline model specification, summarized by equations (25)–(45). In addition, we estimate a version of the model without the unemployment rate as an observable variable. We would like to test whether the unemployment rate contains relevant information for estimation and forecasting. Finally, we assess the fit of the small-scale DSGE model (nested into the baseline specification), which is similar in spirit to the setup presented in Lees *et al.* (2007) and Lubik and Schorfheide (2005). In all cases we compare the performance of the DSGE model with the more flexible DSGE-VAR specification.

Recent literature reports a rather mixed evidence on the comparative performance of structural, reduced-form models and mixed specification such as DSGE-VARs. An important finding of studies by Smets and Wouters (2003, 2007) performed for European and US data, respectively, is that the large-scale New Keynesian DSGE model fits better than unrestricted VAR. Smets and Wouters (2007) demonstrate that only BVAR(4) with Sims and Zha prior can do as well as the DSGE model. Sims (2003) draws attention to a number of shortcomings in Smets and Wouters'

[20] See Del Negro and Schorfheide (2004) and Marcellino and Rychalovska (2012) for technical details on this method.

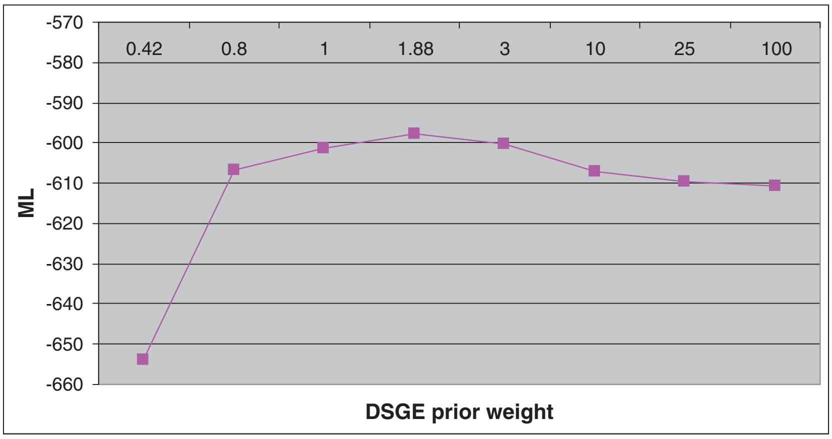

Figure 1. Marginal data density as a function of DSGE prior weight

(2003) analysis, which can potentially lead to over-evaluation of DSGE advantages in terms of the data fit. One of the critical points is related to the use of linearly detrended instead of raw data. The author claims that the data transformation method can distort in- and out-of-sample comparisons. Del Negro *et al.* (2005) address the criticism of Sims, performing a more consistent evaluation exercise based on the original data. More importantly, they apply a new tool for model evaluation, namely the DSGE-VAR approach. Their findings are less favorable for the DSGE model, pointing to a certain degree of model misspecification since the optimal DSGE prior weight is positive but relatively small. Thus relaxing DSGE restrictions significantly improves the model fit. A number of studies evaluate the performance of open-economy DSGE model specifications. In particular, Adolfson *et al.* (2008) test empirical properties and forecasting outcomes of a small open-economy DSGE model with modified UIP condition estimated on Swedish and euro area data. The authors also evaluate the degree of model misspecification combining a VAR(VECM) with a DSGE prior. More specifically, they compare cross-correlation functions for optimal $\tilde{w}$ and $\tilde{w} = \infty$ along with the standard deviations of the variables taken from the VECM covariance matrix. Their results suggest that there are significant differences for real exchange rate autocorrelations and standard deviations, indicating that the model remains misspecified in this direction even with more empirically relevant specification of UIP condition. In addition, they demonstrate that the DSGE-VAR correction does not support the cointegration restrictions in the DSGE model. At the same time, their results suggest that micro-based economic prior is still informative and thus improves marginal likelihood of unrestricted VAR. Lees *et al.* (2007) apply DSGE-VAR methodology to a small open-economy model of New Zealand with explicit inflation target. They assess the DSGE-VAR forecasting performance and use the estimated hybrid structure to identify optimal policy rules. This paper shows that the weight placed on the DSGE prior is significant; both the DSGE and DSGE-VAR model outperform the official forecasts of the Reserve Bank of New Zealand.

Now let us turn to the analysis of the results presented in Table II and see how they contrast with the previous studies. The log data density (LDD) for the DSGE model is higher compared to the DSGE-VAR(2), with the optimal DSGE prior weight being equal to 1.88. This result implies that relaxation of DSGE restrictions via VAR(2) correction does not improve the empirical fit of the model. It should be noted that the value of $\tilde{w}$ cannot be directly compared across different studies. The interpretation of the value of the DSGE-VAR hyperparameter depends on the model size and the size of the dataset. In particular, part of the artificial DSGE observations is 'consumed' in the process of construction of the proper prior distribution[21] and therefore do not count in the actual model evaluation. For example, in our case $\tilde{w}_{\min} \approx 0.42$, whereas the model of Adolfson *et al.* (2007) implies $\tilde{w}_{\min} \approx 2.7$. Thus it is reasonable to consider the 'effective' value of the hyperparameter $\left(\widehat{\tilde{w}} - \tilde{w}_{\min}\right)$ which will measure the number of post-training artificial observations relative to the actual data. Our results imply the optimal weight of 60% on the DSGE model and 40% on the VAR(2). This measure is comparable with previous papers.[22] Analysis of Table II and Figure 1 also provides an idea about how well the VAR(2) approximates the DSGE model. Figure 1 shows the marginal likelihood as a function of the DSGE prior weight. The graph demonstrates that the LDD of the DSGE-VAR with $\tilde{w} = 100(\approx \infty)$ is less than the LDD of the DSGE (see Table II). This result implies that the DSGE model can be approximated by a VAR(2) process only to a limited degree. In other words, the DSGE model embeds a transmission mechanism with greater internal persistence, which translates into a better fit to the data.

[21] Recall that $\tilde{w}_{\min} = (k + n)/T$.
[22] Del Negro *et al.* (2005) and Lees *et al.* (2007) report the optimal weight on DSGE of about 50% and Adolfson *et al.* (2007) 70%.

An approximation error present in our analysis makes it difficult to assess the dimensions in which the DSGE model can be misspecified. In this paper we would like to focus more on the forecast comparison and leave the analysis of the potential model misspecification for further research. However, we believe that the results in Table II support the validity of the DSGE modeling assumptions. Table II also demonstrates that the VAR(2) approximation of the small-scale DSGE model without the labor market block is satisfactory. However, the weight on the DSGE restrictions is lower compared to the baseline specification, at about 45%. Thus the part of the DSGE restrictions associated with the labor market seems to be supported by the data. Modeling labor market dynamics (and rigid wages in particular) substantially adds to the internal propagation mechanism, thus making the DSGE model more in line with the data.

FORECAST EVALUATION AND COMPARISON

Point forecasts

Forecasting performance is an important criterion in the assessment of a model's credibility and usefulness for policy analysis. In this section, we compare the out-of-sample forecast accuracy of the estimated DSGE model and various VARs estimated on the same dataset. In particular, we would like to test whether predictions based on the theoretically grounded DSGE model are competitive with those of reduced-form approaches. Furthermore, by evaluating the outcomes obtained from the models which utilize the prior beliefs, we check whether the prior information plays a role in improving the forecast density and which prior, atheoretical or implied by the DSGE restrictions, has more relevant content for predicting the future dynamics. We calculate forecasts for six macroeconomic time series: output, inflation, real wages, REER, employment and unemployment rate. All the variables except the inflation are in growth rates. The accuracy of the predictions is assessed by using a standard recursive forecast procedure, which implies that the model is estimated up to a certain time period where the forecast distribution from 1 to 8 quarters is computed. The estimation sample is then extended by one more data point. The forecasts are computed for the period 2006:Q1 to 2011:Q3, which gives 23 observations (roughly one-third of

Table III(a). Point forecast accuracy

RMSE	Models				
	AR(2)	VAR(2)	BVAR(2)	DSGE	DSGE-VAR(2)
Output					
1Q	1.6572	1.9784	1.866	**1.5185**	**1.6412**
4Q	1.7595	**1.6482**	1.8726	**1.6726**	1.676
8Q	1.6824	**1.621**	1.8524	**1.6613**	1.6782
Inflation					
1Q	0.4130	0.4259	**0.3986**	**0.4102**	0.408
4Q	**0.3986**	0.4403	**0.4151**	0.4696	0.4834
8Q	**0.3976**	**0.4148**	0.4285	0.5013	0.4985
REER					
1Q	1.1730	1.2542	1.0466	**0.9212**	**0.9059**
4Q	1.2283	1.271	1.081	**0.9692**	**0.9565**
8Q	1.2721	1.177	1.0317	**0.9339**	**0.9404**
Employment					
1Q	**0.2236**	0.2947	0.2573	0.2537	**0.2411**
4Q	0.2893	**0.2795**	**0.2786**	0.480	0.4806
8Q	**0.2851**	**0.2207**	0.347	0.5143	0.4932
Unemployment					
1Q	3.8411	4.3869	**3.6546**	**3.539**	3.9935
4Q	5.2867	6.2366	**3.6665**	**3.933**	4.1593
8Q	4.7127	6.3764	**3.3753**	**4.185**	4.1766
Real wages					
1Q	1.0549	1.2292	0.801	**0.7475**	**0.7753**
4Q	0.9364	0.959	**0.8405**	**0.8382**	0.843
8Q	1.0342	1.075	0.8606	**0.8251**	**0.8342**

Note: All models are estimated on the same dataset, which includes six domestic and three foreign (exogenous) variables. The estimation sample starts in 1995:Q2. The forecast evaluation sample is 2006:Q1–2011:Q3. Bold entries indicate the first- and second-best forecasting model.

Table III(b). Comparing the forecasting performance: robustness analysis

	Models		
1Q, RMSE	VAR(2)	BVAR(2)	DSGE
w/o unemployment data			
Output	1.978	1.889	1.65
Inflation	0.452	0.432	0.418
REER	1.29	1.072	0.934
Employment	0.33	0.277	0.239
Real wages	1.052	0.755	0.821
w/o labor market data			
Output	1.931	1.895	1.567
Inflation	0.435	0.43	0.419
REER	1.222	1.027	0.921

Note: All models are estimated on the same dataset, which includes five (three in the case of estimation without labor market data) domestic and three foreign (exogenous) variables. The estimation sample starts in 1995:Q2. The forecast evaluation sample is 2006:Q1–2011:Q3.

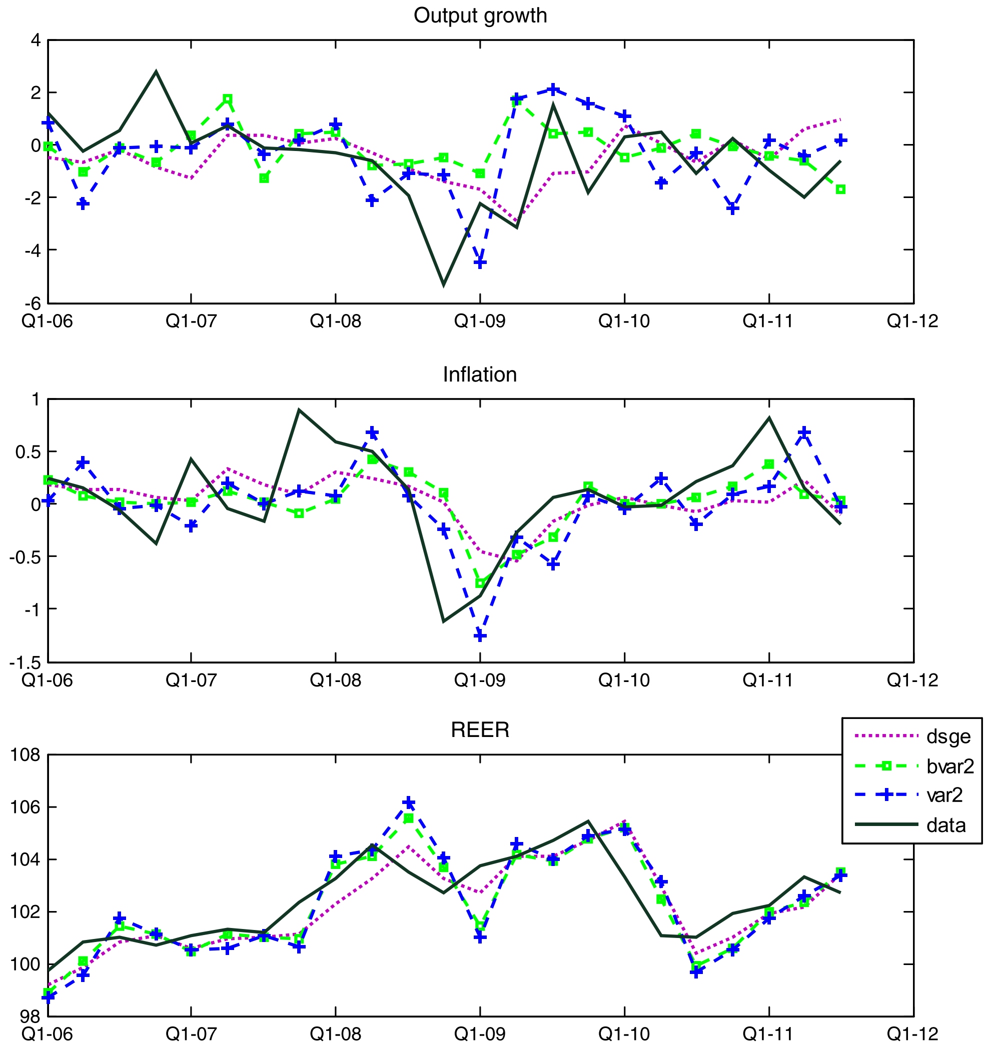

Figure 2. One-quarter forecast comparison

the full sample). All the models are re-estimated every quarter. As a criterion of the forecast accuracy we use a traditional measure—RMSE—which is computed for one-, four- and eight-step-ahead predictions. As a robustness check, we compare 1-quarter-ahead forecasts across different models when a dimension of the observable dataset is reduced. In particular, we check whether our conclusions continue to hold if labor market data are not used in the analysis. The results are presented in Tables III(a) and III(b). Numbers in bold type highlight the first- and second-best performing model in terms of the RMSE. Table III(a) allows the following conclusions to be drawn. First, the DSGE model shows a superior one-step-ahead predictive performance for all the variables except employment. The greatest improvement over the unrestricted VAR is observed for output, REER, unemployment and especially real wages. Over a period up to 2 years the DSGE model forecasting error for output is comparable to that of VAR, whose prediction accuracy improves for the medium-run (4–8 quarters) horizons. Table III(a) also demonstrates that reduced-form models outperform the DSGE in terms of precision of 4-quarter and 8-quarter inflation and employment forecasts. At the same time, the DSGE does considerably better in predicting REER, unemployment and real wages over the longer term. For this data sample, the forecasting performance of the DSGE is not improved by the VAR correction. The BVAR model performs worse in forecasting output but produces more accurate 1-quarter and 4-quarter inflation predictions compared to both VAR and DSGE. Moreover, the BVAR model outperforms both AR and VAR in forecasting unemployment and wages for short- and medium-term horizons. Finally, augmenting the VAR with a theoretical prior based on the DSGE model restrictions significantly improves short-term forecast accuracy for output and delivers a superior exchange rate, unemployment and wages predictions over all the forecast horizons considered here. In addition, a DSGE prior appears to be more informative compared to a Minnesota-style prior when forecasting output and REER, whereas the opposite is true for employment. In predicting wages, the models deliver similar results. As for the robustness check, the DSGE compares to the (B)VAR equally well in smaller-scale specifications, which do not include unemployment and labor market data (see Table III(b)). Comparison of the results presented in the Tables III(a) and III(b) indicates that using unemployment as an observable variable brings the most notable improvements in forecasting the output and wage dynamics.

A visual demonstration of the forecasting performance is shown in Figures 2 and 3, which present 1-quarter forecast comparison across alternative models. These plots are useful because they enable us to evaluate which models did a better job in predicting the most recent financial crisis event. The graphs show that VAR predictions are generally more volatile. In particular, this model predicts a sharp decline in the output growth around 2009:Q1, followed by

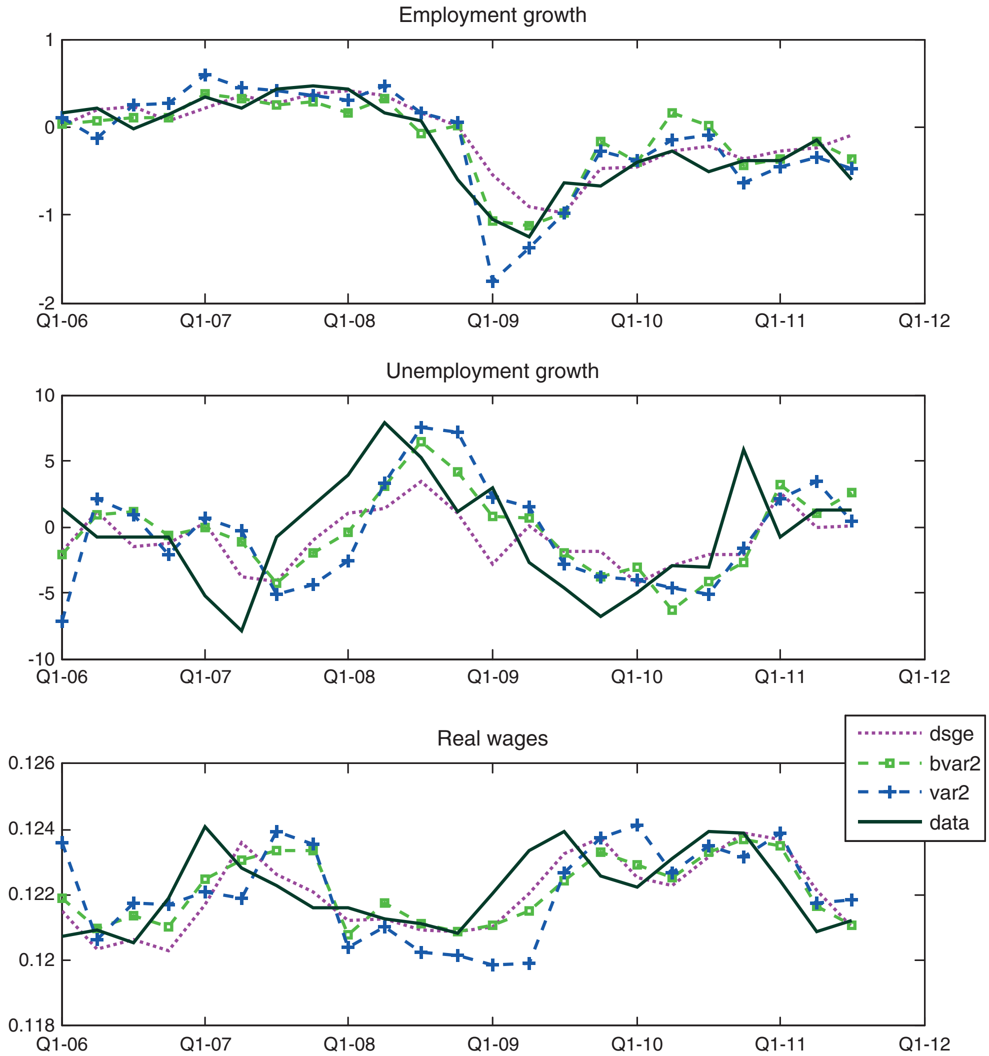

Figure 3. One-quarter forecast comparison

a quick recovery. The VAR overpredicts the decrease in inflation, employment and wages and also overestimates the growth of the unemployment rate after the financial distress. DSGE predictions show more persistent evolution of real variables, followed by a slower recovery. Thus qualitative characteristics of DSGE-produced forecasts better comply with the observed dynamics. BVAR models generate most accurate predictions (in terms of magnitude and persistence) for inflation and employment decline during this period. At the same time, BVAR fails to forecast a pronounced drop in the output growth. BVAR's predictions for real wages and unemployment are close to that of the DSGE.

Overall, the analysis presented here demonstrates that DSGE forecasts can compete well with more empirical models. The results of this section agree well with the conclusions from other recent studies that evaluate the ability of structural models to represent a viable alternative to reduced-form specifications in forecasting experiments. In particular, Adolfson *et al.* (2008) report that a DSGE small open economy model developed for Sweden appears to be the best forecasting tool out of different (including VARs) models they compare. Smets and Wouters (2003, 2007) confirm the good forecast performance of the DSGE model relative to the VAR and BVAR. Lees *et al.* (2007) also emphasize a competitive performance of DSGE and DSGE-VAR in forecasting the dynamics of the New Zealand economy. For their sample, the BVAR with Minnesota prior shows the best predictive accuracy.

Density forecasts

In the previous subsection, we compared the alternative models in terms of their point forecast ability. Another important measure of forecasting performance is the comparison of predictive densities, which enables evaluation of the accuracy of forecasts by taking into account forecast uncertainty. The evaluation and ranking of density forecasts can be done by comparing the log predictive density scores (LPDS), as described in Adolfson *et al.* (2007) and Christoffel *et al.* (2010). Under the assumption that h-step-ahead predictive density is normally distributed, the LPDS for variable i can be written as

$$s_t\left(y^i_{t+h}\right) = -0.5\left[\log(2\pi) + \log\left(V^i_{t+h/t}\right) + \left(y^i_{t+h} - \overline{y}^i_{t+h/t}\right)^2 / V^i_{t+h/t}\right]$$

Table III(c). Density forecast accuracy

	Models			
Score	VAR(2)	BVAR(2)	DSGE	DSGE-VAR(2)
Output				
1Q	−2.3096	−2.3455	**−1.8574**	**−1.9377**
4Q	**−1.9425**	−2.1455	−1.951	**−1.9437**
8Q	−1.9476	−2.1227	**−1.9388**	**−1.9304**
Inflation				
1Q	−1.6526	−1.028	**−0.6937**	**−0.9341**
4Q	−1.0384	**−0.8669**	**−0.8928**	−1.2207
8Q	**−0.6647**	**−0.9126**	−0.9523	−1.2941
REER				
1Q	−3.0939	−1.8552	**−1.3365**	**−1.3882**
4Q	−2.1563	−1.6497	**−1.3912**	**−1.4817**
8Q	−1.7591	−1.5579	**−1.3655**	**−1.4734**
Employment				
1Q	−0.2	**−0.0767**	−0.2	**−0.1322**
4Q	**−0.2952**	**−0.2456**	−0.7317	−0.7455
8Q	**−0.3087**	**−0.3945**	−0.79	−0.7615
Unemployment				
1Q	−2.9121	**−2.722**	**−2.7929**	−2.8178
4Q	−3.2762	**−2.803**	−2.8656	**−2.8500**
8Q	−3.2734	**−2.7626**	−2.9188	**−2.8682**
Real wages				
1Q	−1.8865	**−1.2025**	−1.2540	**−1.1778**
4Q	−1.4052	**−1.2601**	−1.3115	**−1.2495**
8Q	−1.5155	**−1.2801**	−1.3095	**−1.2355**

Note: All models are estimated on the same dataset, which includes six domestic and three foreign (exogenous) variables. The estimation sample starts in 1995:Q2. The forecast evaluation sample is 2006:Q1–2011:Q3. Bold numbers indicate the first- and second-best forecasting model.

where $\overline{y}^{i}_{t+h/t}$ and $V^{i}_{t+h/t}$ are the posterior mean and variance of h-step-ahead simulated forecast distribution for variable i. The average score in forecasting variable i with the model m is given by

$$\text{Score}^{m}_{i,h} = T_h^{-1} \sum_{t=T}^{T+T_h-1} s_t \left(y^{i}_{t+h}\right)$$

where T_h denotes the number of $h-$step-ahead forecasts. It should be noted that the predictive density of the DSGE model estimated with Bayesian methods does not have a known analytical form. Following Adolfson *et al.* (2007) we will use the multivariate normal approximation of the DSGE predictive density. This assumption is convenient because of the property of the multivariate normal density that the distribution of any subset of variables is also normal. Christoffel *et al.* (2010) point out that, for models estimated with Bayesian methods, the only source of non-normality of the predictive density is the parameter uncertainty. Since normally only a small fraction of the forecast error variance is attributed to the parameter uncertainty, the normality assumption does not involve significant misspecification in computation of the log predictive score. Table III(c)(c) reports the average log predictive scores in forecasting the endogenous variables from 1 to 8 steps ahead. Analyzing this measure of the accuracy of the predictions, we can see that DSGE(-based) models have significantly better forecast density for output and inflation at shorter horizon. At longer horizons, the reduced-form (VAR) and structural models deliver similar predictive score for output, while for inflation and employment the VAR model outperforms the DSGE. The LPDS also suggests a superior performance of the DSGE model in terms of the forecast density for REER, unemployment and real wages at all considered forecast horizons. BVAR is particularly successful in terms of the Score in predicting employment, unemployment and real wages.

CONCLUSIONS

In this paper we develop and estimate a DSGE model for Luxembourg, as an example of a small open economy within the single currency area. We allow for a sufficiently rich specification which enables us to include unemployment as well as open-economy variables such as the real exchange rate in the estimation procedure, along with the standard macroeconomic and labor market indicators. The model contains a set of frictions and structural shocks typically used in the DSGE literature. We demonstrate that the estimated DSGE model is relatively well identified, has good data fit and reasonably estimated parameters. In addition, the model shows a competitive forecasting performance (in terms of both point and density) compared to reduced-form models such as VARs. In this respect, our results are in line with the conclusions reached in previous studies that the new generation of DSGE models no longer faces the tension between rigor and fit. In particular, we illustrate that the DSGE model produces sizable (one-step-ahead) forecasting gains in terms of RMSE and Score over the unrestricted VAR, especially for such variables as GDP, real exchange rate, unemployment and real wages. The predictions stay competitive at longer forecasting horizons.

As a result of a sufficiently rich specification, the solution to the model implies rather tight cross-equation restrictions on the estimated structure. On the one hand, this can be considered as a limitation of the approach. On the other hand, micro-founded restrictions that have a realistic content can bring useful additional information to the estimation procedure and thus improve the model fit. In particular, the DSGE-VAR analysis demonstrates that the optimal weight on the DSGE restrictions is significant (about 60%) and the VAR(2) correction is not helpful in improving the DSGE model fit. At the same time, the DSGE-based prior significantly improves the short-term forecast accuracy of the unrestricted VAR for output, and also determines a superior performance of the DSGE-VAR model in predicting exchange rate, unemployment and wages over all the forecast horizons considered here. When compared to an atheoretical Minnesota-style prior, the DSGE restrictions appears to be more useful in forecasting output and REER, whereas the opposite is true for employment. The results of this analysis do not imply, of course, the absence of model misspecification but at the same time they show that a DSGE structure provides a reasonable approximation of the reality.

In addition, we would like to note a number of caveats that could potentially affect the validity of our results. First, we admit that the evaluation of the model on the relatively short data sample available for Luxembourg (66 observations) can lead to overestimation of the performance of the prior-based specifications. Secondly, the forecasting performance is conditional on the prior knowledge of the (exogenous) foreign variables. In practice, it is extremely difficult to correctly anticipate the foreign dynamics and its potential impact on the small open economy. Therefore, the forecast errors may change significantly depending on the assumptions on the foreign variables. Finally, the forecasting performance may also be flattered by the fact that we are using revised rather than real-time data.

As a final point, we would like to discuss possible extensions. In particular, it would be useful to extend the model by considering a more disaggregated structure and, in particular, incorporate the financial services sector, which constitutes a significant portion of the Luxembourg economy and can be a driving force of the economy as a whole. Since the responses of this sector to monetary and other shocks might be quite specific, the overall characteristics and model predictions can be affected. In addition, the properties of the Luxembourg economy differ significantly from the rest of the EMU. Therefore, it would make sense to improve the existing specification by modeling heterogeneous features of both regions other than the size and degree of openness (for example, we could allow for different growth rates and provide more elaborate modeling of the EMU with individual parametrization and better identification of area-wide shocks).

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APPENDIX

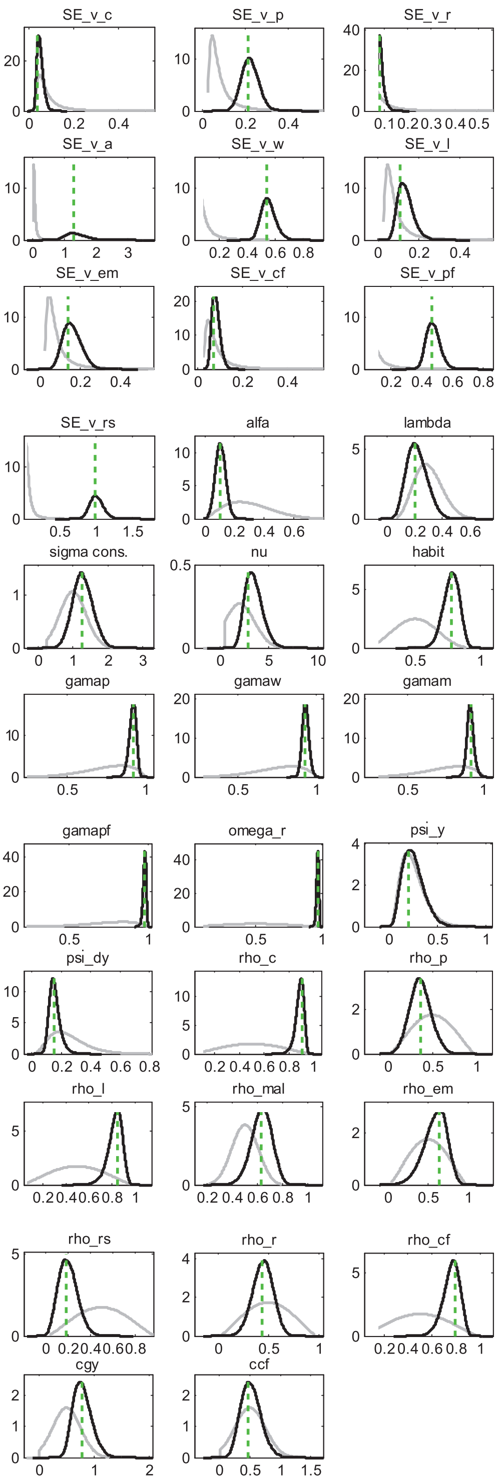

Figure A.I. Priors and posteriors

Table A.I. Comparison of the posterior distribution of DSGE structural parameters for alternative estimation samples

Parameter		Prior distribution			Posterior distribution			
					1995:Q1–2007:Q4		1995:Q1–2011:Q3	
		Type	Mean	SD	Mode	SD	Mode	SD
Production function	λ	Beta	0.3	0.1	0.223	0.087	0.202	0.077
Degree of openness	α	Beta	0.3	0.15	0.098	0.039	0.102	0.034
Consumption utility	σ_c	Norm	1	0.375	1.370	0.297	1.256	0.292
Labor utility	η	Norm	2	1.5	2.303	0.737	2.873	0.804
Consumption habit	χ	Beta	0.5	0.15	0.701	0.095	0.776	0.062
Calvo prices	γ^p	Beta	0.75	0.15	0.929	0.023	0.923	0.022
Calvo wages	γ^w	Beta	0.75	0.15	0.939	0.023	0.929	0.019
Calvo employment	γ^m	Beta	0.75	0.15	0.929	0.028	0.918	0.021
Calvo foreign prices	γ^{p*}	Beta	0.75	0.15	0.986	0.009	0.977	0.01
Policy rule: lagged int.rate	ω_r	Beta	0.5	0.2	0.975	0.010	0.973	0.010
Policy rule: output	ψ_y	Gamma	0.25	0.125	0.220	0.111	0.201	0.101
Policy rule: lagged output	$\psi_{\Delta y}$	Gamma	0.25	0.125	0.183	0.051	0.151	0.034

Table A.II. Comparison of the posterior distribution of DSGE shock processes for alternative estimation samples

Parameter		Prior distribution			Posterior distribution			
					1995:Q1–2007:Q4		1995:Q1–2011:Q3	
		Type	Mean	SD	Mode	SD	Mode	SD
Standard deviations								
Consumption preference	υ_c	Inverse gamma	0.1	2	0.043	0.014	0.037	0.01
Productivity	υ_a	Inverse gamma	0.1	2	1.197	0.315	1.296	0.306
Price markup	υ_p	Inverse gamma	0.1	2	0.225	0.036	0.212	0.038
Wage markup	υ_w	Inverse gamma	0.1	2	0.583	0.059	0.54	0.049
Relative price	υ_{rs}	Inverse gamma	0.1	2	0.905	0.092	0.985	0.088
Labor supply	υ_l	Inverse gamma	0.1	2	0.089	0.032	0.108	0.033
Exogenous employment	υ_{em}	Inverse gamma	0.1	2	0.135	0.038	0.142	0.042
Foreign demand	υ_{c*}	Inverse gamma	0.1	2	0.054	0.015	0.071	0.017
Foreign prices	υ_{p*}	Inverse gamma	0.1	2	0.374	0.038	0.463	0.042
Interest rate	υ_r	Inverse gamma	0.1	2	0.075	0.013	0.08	0.011
Persistence and correlation								
Consumption	ρ_c	Beta	0.5	0.2	0.910	0.031	0.909	0.024
Price markup	ρ_p	Beta	0.5	0.2	0.235	0.133	0.368	0.122
Relative price	ρ_{rs}	Beta	0.5	0.2	0.173	0.094	0.184	0.087
Labor supply: AR	ρ_l	Beta	0.5	0.2	0.876	0.051	0.85	0.055
Labor supply: MA	$\rho_{ma,l}$	Beta	0.5	0.1	0.629	0.082	0.631	0.079
Exogenous employment	ρ_{em}	Beta	0.5	0.2	0.670	0.113	0.635	0.134
Interest rate	ρ_r	Beta	0.5	0.2	0.465	0.101	0.438	0.101
Foreign demand	ρ_{c*}	Beta	0.5	0.2	0.785	0.089	0.789	0.068
Demand–productivity	ρ_{ag}	Norm	0.5	0.25	0.834	0.198	0.785	0.173
Consumption–foreign demand	ρ_{cf}	Norm	0.5	0.25	0.430	0.194	0.468	0.160

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