

On page 28 of Fernandez-Villaverde and Rubio-Ramirez we are told that equation (4) can be written as below, after replacing a number of terms with ‘aux’.

$$aux \exp^{\widehat{aux}_t} - h\beta z \mathbb{E}_t \exp^{(\widehat{aux}_{t+1} + \widehat{z}_{t+1})} = \tilde{\lambda} e^{\hat{\lambda}_t} \quad (1)$$

This equation is then log-linearized as:

$$aux(\widehat{aux}_t - h\beta z \mathbb{E}_t(\widehat{aux}_{t+1} + \widehat{z}_{t+1})) = \tilde{\lambda} \hat{\lambda}_t \quad (2)$$

I try to reach this result using a simple Taylor series expansion based on the following process (source: <http://www.tau.ac.il/~razin/loglinearization.pdf>).

First, define $X_t = X(X_t/X) = X \exp^{\log(X_t/X)} = X \exp^{x_t}$

Taking a first order Taylor series expansion around the steady state provides:

$$X \exp^{x_t} \approx X \exp^0 + X \exp^0(x_t - 0) \approx X(1 + x_t) \quad (3)$$

Applying this to the first equation in this document, we get:

$$aux((1 + \widehat{aux}_t) - h\beta z \mathbb{E}_t[(1 + \widehat{aux}_{t+1}) + z(1 + \widehat{z}_{t+1})]) = \tilde{\lambda}(1 + \hat{\lambda}_t) \quad (4)$$

How does this simplify to the second equation (equation 5 in Fernandez-Villaverde and Rubio-Ramirez (2006))? Am I making a mistake somewhere or not understanding something correctly, or is there another procedure that I’m not aware of that they use?