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# Long-run productivity risk: A new hope for production-based asset pricing? ☆

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ABSTRACT

The examination of the intertemporal distribution of US productivity risk suggests that the conditional mean of productivity growth is an important determinant of macro quantities and asset prices. After establishing this empirical link, I rationalize it in a production economy featuring long-run productivity risk, Epstein and Zin (1989) preferences, and investment frictions. Both convex capital adjustment costs and convex reallocation costs across consumption and investment produce an annual equity premium as sizeable as in the data.



## 1. Introduction

Aggregate productivity is one of the most important indicators of both the macroeconomic and financial conditions of a country. Fama (1981, 1990), Cochrane (1996), and Balvers and Huang (2007), among others, have already documented the existence of a relevant link between the movement in asset prices and real economic activity at business cycle frequency. All of these studies, however, share one common feature: they do not distinguish the specific impacts that different sources of productivity uncertainty can have on stock prices. In particular, they do not disentangle the role of predictable fluctuations that are active at low frequency and that explain productivity growth rate swings over long horizons. In this paper, this type of fluctuation is referred to as long-run productivity risk.

This paper investigates the possibility that long-run productivity risk is the original source of both the long-run consumption and dividend risks studied by Bansal and Yaron (2004). The first step of my analysis documents the existence of a predictable component in US productivity growth that affects both aggregate stock market prices and major macroeconomic variables. The second step of this study consists in proposing a novel production-based dynamic stochastic general equilibrium (DSGE) model with long-run productivity shocks that produces improved asset price implications.

Working with aggregate productivity data, I show that the conditional mean of annual productivity growth is time-varying and extremely persistent. Most importantly, productivity news is tightly related to both stock market and interest rate fluctuations. These results are robust across different identification schemes and different productivity measures. In addition, the long-run component in productivity also has a strong economic significance: consumption, investment, and

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output all show a statistically significant positive exposure to the long-run productivity component. Together, short-run and long-run productivity shocks explain up to 70% of the volatility of the macroeconomic variables mentioned above.

After demonstrating an empirical link between productivity, asset prices, and macroeconomic variables, this study develops a production-based asset model with complete markets featuring long-run uncertainty about the productivity growth rate. The main goal of this second step is to explore in detail the theoretical implications of productivity growth predictability (long-run productivity risk) in the context of a model with endogenous investment and recursive utility. This theoretical approach bridges part of the gap between the current long-run risk literature and the macroeconomic literature by proposing a workhorse framework in which to study the co-movements of asset prices and quantities simultaneously over both the short and the long horizon.

The model is extremely parsimonious. Productivity growth is exogenous and affected by two different sources of uncertainty: a short-run shock that is *i.i.d.* (standard in the real business cycle [RBC] literature), and a long-run component that is responsible for small but persistent fluctuations in the drift of productivity. The representative consumer has Epstein and Zin (1989) and Weil (1989) preferences. These preferences disentangle the intertemporal elasticity of substitution (IES) from the relative risk aversion coefficient (RRA) and are sensitive to the intertemporal distribution of risk. Capital accumulation is subject to Jermann (1998) convex adjustment costs so that the supply curve of new capital is not perfectly flexible. As a result, the price of new capital is time-varying and increases when the economy seeks to increase aggregate investment, consistent with standard q-theory (see, among others, Zhang, 2005; Liu et al., 2009; Li and Zhang, 2010).

Under this setup, the model successfully reproduces key features of both asset prices and macroeconomic quantities such as consumption, investment, and output. This is significant because the reconciliation of asset market factors with aggregate quantities behavior has proved a challenge for modern DSGE models (see, among others, Rouwenhorst, 1995; Jermann, 1998; Lettau and Uhlig, 2000; Boldrin et al., 2001; Cochrane, 2005). In particular, the model presented here produces a sizable equity premium and a low average risk-free rate, with a moderate amount of RRA and an IES slightly larger than one. While in common production economies the premium is generated through an unrealistically high contemporaneous correlation between returns and consumption growth, in my model this correlation is very low, as in the data. Furthermore, the model delivers an implied risk-free rate as persistent as that observed in the data, while the excess returns are in contrast almost unpredictable, again consistent with the empirical evidence.

Although these results look very promising, it has to be acknowledged that the returns volatility puzzle remains partially unexplained in my framework. Embodied in the model is a relevant trade-off between the volatility of the quantities and that of asset returns. This trade-off is generated by the need for a real friction that allows for time variation in the marginal price of capital. The more severe the friction, the less volatile the quantities. On the other hand, the weaker the friction, the smoother the equity returns. In order to match the volatility of the quantities, the adjustment costs are calibrated to be very small. The introduction of the long-run component, however, yields non-negligible fluctuations in the stock price even with a very mild friction. The long-run shocks, in fact, are able to produce substantial shifts in the demand of new capital and generate relevant price movements even if the new capital supply curve is not very steep.

The performance of the model in regard to second moments improves substantially when we focus on contemporaneous correlations. The model is able to produce the right amount of co-movement not only between returns and real variables, but also across the macro variables. A well-known problem with standard RBC models is that they produce an almost perfect correlation between consumption and investment. In the data, however, the contemporaneous correlation between consumption and investment growth is about 45% at quarterly frequencies in the post-World War II period and about 39% at an annual frequency when pre-World War II data are included. Under the benchmark calibration, when the IES is sufficiently high the substitution effect dominates the income effect for the long run: good news for the expected productivity growth rate provides a strong incentive to reduce consumption and invest more. Meanwhile, good news for the short run implies an increase in both consumption and investment. The overall correlation between consumption and investment is positive but moderate, in line with the data.

To my knowledge, this paper is one of the first to study the interaction between predictability in productivity growth and recursive preferences. The results in this study show that the IES has an extremely important impact on the dynamics of both quantities and prices, mostly in response to long-run shocks. The RRA, in contrast, has only a marginal impact on quantities, as in Tallarini (2000).

The present study also proposes a novel analysis of the role of different frictions in capital accumulation. Specifically, I compare my benchmark specification with Jermann's adjustment costs to a specification in which there is no adjustment cost, and to one that adopts the time-friction of Boldrin et al. (2001) by assuming that both labor and investment decisions are determined before the realization of productivity shocks. This experiment is particularly interesting because it relates asset price implications to very different consumption smoothing possibilities.

Three relevant results emerge from this study. First, a real friction on investment is absolutely required in order to allow the Bansal and Yaron (2004) results to hold also in a production economy. Second, in an economy in which both labor and capital allocations across the consumption and the investment sectors must be decided one period in advance (Boldrin et al., 2001), the asset pricing results deteriorate substantially. This is because a one-period delay in the adjustment of investment plays no relevant role with respect to news about productivity gains or losses to be realized in the long-run. A comparison of impulse responses with respect to long-run news shows that the behavior of the model with the Boldrin et al. (2001) time-friction closely resembles that of the model without adjustment costs, and hence it fails to expose the equity market to long-run risk.

The third result consists in showing that the replacement of the Boldrin et al. (2001) time-friction with a convex reallocation cost produces positive asset pricing results, comparable to those obtained with capital adjustment costs. Specifically, when resources can immediately be reallocated across the consumption and investment sectors by paying a convex cost, the value of capital is increasing in the investment-output ratio, consistent with US annual data. Therefore, upon the arrival of positive long-run news, the agent's incentive to reallocate resources toward investment increases the shadow value of capital and produces a capital gain. At the equilibrium, the positive exposure of the aggregate excess returns to long-run risk produces an annual equity premium of 4.1%.

The next section discusses closely related articles. Section 2 presents the empirical findings on long-run productivity risk. The model and main asset pricing results are presented in Section 3. Section 4 examines the sensitivity of the results to (i) the elasticity of substitution between consumption and leisure; (ii) different specifications of long-run risk; (iii) the decision horizon of the representative agent; and (iv) the presence of volatility shocks. Section 5 concludes.

### 1.1. Literature discussion

This paper is intended as a contribution to both the long-run risk literature and the literature regarding production-based asset pricing. For the sake of brevity, I discuss only a subset of papers that are closely related to mine, beginning with Tallarini (2000). Tallarini is the first to focus on an RBC model with recursive preferences. He examines a completely frictionless production economy with an IES constrained to be one. In two independent and contemporaneous papers, both Kaltenbrunner and Lochstoer (2010) and Campanale et al. (2010) work with a production economy with convex capital adjustment costs and Epstein and Zin (1989) preferences.[1]

All these authors work with just one source of productivity uncertainty, and they do not estimate long-run productivity risk. Moreover, these studies do not solve the equity premium puzzle, as they target the equity Sharpe ratio rather than the equity premium. However, as the present paper shows, by neglecting long-run shocks researchers may miss two important elements. First, the long-run shocks provide crucial information on the role of the IES in determining the co-movements of investment, consumption, and returns. Second, neglecting the long-run uncertainty channel may lead to substantial underestimation of the true market price of risk.

Jaimovich and Rebelo (2009) examine the role of productivity news for the determination of both aggregate and sectoral co-movements in the context of a model with investment-specific shocks, endogenous capital utilization, Greenwood et al. (1988) preferences, and investment adjustment costs. The present study differs from Jaimovich and Rebelo (2009) in several respects. First, Jaimovich and Rebelo (2009) adopt time-additive preferences with a low IES and abstract away from asset pricing considerations, the main focus in this manuscript. Second, Jaimovich and Rebelo (2009) focus mainly on a setting in which future sector-specific productivity is revealed in advance with certainty. My long-run productivity risk news, instead, capture shocks to conditional expected productivity and leaves the agent exposed to ex-post variations in aggregate productivity growth. Third, when considering the arrival of noisy news about future sector-specific productivity, Jaimovich and Rebelo (2009) focus on a calibration with time-varying uncertainty. My main results are obtained in a setting with constant volatility.

The approach proposed in this paper has attracted significant attention. Kuehn (2008) proposes a model with precommitted investment. Ai and Kiku (2013) extend the analysis in this manuscript to study the accumulation of growth options and the value premium. Gourio (2012) adopts a similar production setting to study the role of rare events, Ready (2013) introduces an oil sector to account for oil futures prices, Jahan-Parvar and Liu (2013) explore long-run productivity risk in the context of ambiguity, and Backus et al. (2010) focus on business cycle leads and lags.

My empirical investigation is related to the literature on macroeconomic news (see, among others, Beaudry and Portier, 2006). My empirical analysis differs from that of Beaudry and Portier (2006) in at least two respects. First, it adopts a different econometric specification that allows for better testing of the implications of long-run risk models. Second, the empirical results suggest that consumption and investment may have opposite responses to long-run news. This result, however, depends on whether the productivity measure adopted in the estimation is utilization-adjusted (Basu et al., 2006). Additionally, a fully specified DSGE model is used in order to explicitly study the role of the deeper parameters of the model. The model yields a positive link between investment and macroeconomic news, consistent with Beaudry and Portier (2006). My economy, however, is not designed to produce consumption and investment co-movements upon the arrival of long-run news. Frictions as in Jaimovich and Rebelo (2009), Beaudry and Portier (2012), Favilukis and Lin (2011), and Kuehn et al. (2010) could resolve this problem, but a study of the relevance of these frictions is left for future work.

## 2. Empirical evidence

Let $A_t$ denote the level of productivity at time $t$ and lowercase letters denote log-units. The growth rate of productivity is decomposed as follows:

$$\Delta a_{t+1} = \mu + \underbrace{x_t}_{LRR} + \underbrace{\sigma\epsilon_{a,t+1}}_{SRR}$$
$$x_t = \rho x_{t-1} + \sigma_x \epsilon_{x,t},$$

[1] My analysis of the long-run productivity risk in the context of a production economy was begun in 2005. The current manuscript is based on my job market paper "Welfare Costs and Long-Run Consumption Risk in a Production Economy," presented for the first time at the E.C.B. Macro Seminars, August 2005.

$$\begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} \sim iidN\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xa} \\ \rho_{xa} & 1 \end{bmatrix} \right), \tag{1}$$

where $x$ refers to the long-run risk (LRR) component in productivity growth, and $\epsilon_a$ is short-run growth risk (SRR). I study the intertemporal distribution of productivity risk implied by four different econometric specifications of the long-run component. A detailed explanation of the relevance of these representations follows.

First of all, I follow Nelson and Plosser (1982) in assuming that productivity growth follows an AR(1) process,

$$\Delta a_{t+1} = \mu(1-\rho) + \rho\Delta a_t + \epsilon_{a,t+1}.$$

This is equivalent to imposing $\sigma_x = \rho\sigma$ and $\rho_{xa} = 1$ in Eq. (1). Under this specification, the parameter $\rho$ simultaneously determines both the persistence and the magnitude of the long-run shocks. Since the extent of predictability in productivity growth is moderate, this specification tends to underestimate the half-life of the long-run news. This problem can be solved by looking at an ARMA(1,1) representation of productivity growth,

$$\Delta a_{t+1} = \mu(1-\rho) + \rho\Delta a_t - b\epsilon_{a,t} + \epsilon_{a,t+1},$$

which corresponds to imposing the restrictions $\rho_{xa} = 1$ and $\sigma_x = (\rho - b)\sigma$ in Eq. (1). In this case, the half-life of the long-run news is determined by $\rho$, whereas the magnitude of these shocks depends on the difference between the AR and the MA coefficients, $\rho - b$. An ARMA(1,1) with roots close to unity is hence able to capture long-lasting news shocks without implying excessive predictability in productivity growth, $\Delta a$.

Both the AR(1) and the ARMA(1,1) are univariate processes and are not able to disentangle short- and long-run news, as both specifications impose $\rho_{xa} = 1$. In order to allow for $\rho_{xa} \neq 1$, I also consider two multivariate approaches. Under the first approach, Eq. (1) is estimated jointly with the following equation:

$$pd_{t+1} = \rho_{pd} pd_t + \alpha_1 \sigma_x \epsilon_{x,t+1} + \alpha_2 \sigma \epsilon_{a,t+1},$$

where $pd$ denotes the demeaned log price-dividend ratio, and $\alpha_1$ and $\alpha_2$ denote exposure coefficients. In this case, two observable variables ($\Delta a$ and $pd$) are used to filter short- and long-run shocks ($\epsilon_a$ and $\epsilon_x$). Specifically, after initializing the long-run component at zero, for a given set of parameters $\{\rho, \rho_{pd}, \alpha_1, \alpha_2\}$ I can recursively recover the entire time series of both the short-run and the long-run news and can also jointly estimate the variances and the correlations of the two shocks, $\{\sigma, \sigma_x, \rho_{xa}\}$, together with all the other structural parameters. This approach is denoted as the bivariate (BIV) estimation.

My second, multivariate approach follows Bansal et al. (2010) (henceforth BKY). BKY identify long-run consumption risk through a forecasting procedure with multiple predictive variables. Similarly to them, the long-run component in productivity is identified by projecting one-period-ahead productivity growth, $\Delta a_{t+1}$, on the current risk-free interest rate, $r_t^f$, and price-dividend ratio, $pd_t$:

$$\Delta a_{t+1} = \mu + \underbrace{r_t^f \beta_{rf} + pd_t \beta_{pd}}_{x_t} + \epsilon_{a,t+1}.$$

The short-run shocks are identified using the residuals of the regression above, $\epsilon_{a,t+1}$, and the long-run news is obtained by estimating the following AR(1) process:

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_{x,t}.$$

In this case, the persistence of the long-run component, $\rho$, is a convolution of the persistence of the risk-free rate and the price-dividend ratio.

Summarizing, I estimate Eq. (1) using the following four representations for the long-run component:

$$x_t = \begin{cases} \rho x_{t-1} + \rho\sigma\epsilon_{a,t}, & AR(1) \\ \rho x_{t-1} + (\rho - b)\sigma\epsilon_{a,t}, & ARMA(1,1) \\ \rho x_{t-1} + (pd_t - \rho_{pd} pd_{t-1} - \alpha_2 \sigma \epsilon_{a,t}) \dfrac{1}{\alpha_1 \sigma_x} & BIV \\ r_t^f \beta_{rf} + pd_t \beta_{pd} & BKY \end{cases}, \tag{2}$$

and report the results for three different measures of productivity in Table 1.

The first column refers to results obtained using Solow residuals for the US economy over the long sample 1930–2008. The construction of these residual is standard (details in the appendix). The results in the second column are based on a multifactor productivity index that measures the value-added output per combined unit of labor and capital input in private business and private nonfarm business. This is a benchmark economic indicator for the US economy provided by the Bureau of Labor Statistics (BLS).[2] The third column focuses on the utilization-adjusted productivity measure proposed by Basu et al. (2006) and provided by the San Francisco Fed.[3]

[2] The index is available at ftp://ftp.bls.gov/pub/special.requests/opt/mp/prod3.mfptablehis.zip.
[3] This index is available at http://www.frbsf.org/csip/tfp.php.

**Table 1**
Predictability in productivity growth.

| Productivity sample | Statistics | Solow 1930–2008 | BLS 1950–2008 | Basu et al. 1950–2008 |
|---|---|---|---|---|
| AR(1) model | $\rho$ | 0.487*** | 0.065 | 0.182 |
| | | (0.094) | (0.123) | (0.129) |
| | $R^2$ | 0.237 | 0.06 | 0.03 |
| | $\beta_{\epsilon_a\vert\epsilon_{pd}}$ | 3.481*** | 4.17*** | −1.133 |
| | | (0.939) | (1.082) | (1.532) |
| ARMA(1,1) model | $\rho$ | 0.206 | 0.89*** | 0.906*** |
| | | (0.190) | (0.048) | (0.108) |
| | $b$ | −0.45* | 0.98*** | 0.869 *** |
| | | (0.23) | (0.032) | (0.140) |
| | $R^2$ | 0.32 | 0.12 | 0.07 |
| | $\beta_{\epsilon_a\vert\epsilon_{pd}}$ | 3.594*** | 4.380*** | −0.942 |
| | | (0.992) | (1.156) | (1.574) |
| | $p$-val for $H_0: b=0$ | 0.058 | 0.000 | 0.000 |
| Bivariate model | $\rho$ | 0.766*** | 0.991*** | 0.842*** |
| | | (0.032) | (0.016) | (0.095) |
| | $\sigma_x/\sigma$ | 0.320*** | 0.048*** | 0.039** |
| | | (0.067) | (0.016) | (0.015) |
| | $\rho_{xa}$ | −0.014*** | 0.015*** | 0.012*** |
| | | (0.000) | (0.000) | (0.002) |
| | $\alpha_2$ | 109.77*** | 341.11*** | 50.06*** |
| | | (11.061) | (62.23) | (12.73) |
| BKY model | $\rho$ | 0.714*** | 0.656*** | 0.675*** |
| | | (0.081) | (0.087) | (0.085) |
| | $\sigma_x/\sigma$ | 0.295*** | 0.136*** | 0.109*** |
| | | (0.057) | (0.019) | (0.015) |
| | $\rho_{xa}$ | −0.001 | −0.000 | −0.000 |
| | | (0.004) | (0.003) | (0.002) |

This table summarizes the estimation results for the productivity processes specified in Eqs. (1) and (2). The coefficient $\beta_{\epsilon a|\epsilon_{pd}}$ is specified in Eq. (3). Numbers in parentheses are Newey–West adjusted standard errors. Data are annual and their sources are reported in the Appendix. *Solow* refers to a measure of productivity based on Solow residuals. *BLS* refers to multifactor-adjusted productivity computed by the Bureau of Labor and Statistics. *Basu et al.* refer to the Basu et al. (2006) measure of utilization-adjusted productivity provided by the San Francisco Fed.

The Basu et al. (2006) and BLS productivity measures are only available for the post-World War II period. The sample for all three productivity measures ends in 2008 to prevent the results from being driven by the Great Recession. Data are annual for two reasons. First, the data are not altered by any seasonal adjustment; second, they are more likely to contain a better signal and less noise related to the low-frequency component of productivity (see, among others, Bansal et al., 2010; Colacito and Croce, 2011).

The top panel shows the estimation results for the AR(1) process. Under this specification, the persistence of the predictable component is moderate across all productivity measures, and it is statistically significant only for the Solow residuals case. This is not surprising, given the small extent of predictability in productivity growth, as measured by the $R^2$ of these regressions. To study the connection between productivity and equity value, I also identify the price-dividend news, $\epsilon_{pd,t}$, as the residuals of the following equation:

$$pd_t = \beta_{0,pd} + \rho_{pd} pd_{t-1} + \epsilon_{pd,t},$$

and look at their contemporaneous exposure, $\beta_{\epsilon_a|\epsilon_{pd}}$, to productivity news[4]:

$$\epsilon_{pd,t} = \beta_0 + \beta_{\epsilon_a|\epsilon_{pd}} \epsilon_{a,t} + \epsilon_t. \tag{3}$$

This correlation is positive and statistically significant for both the Solow residual and the BLS measure, suggesting the existence of a positive link between productivity growth news and price-dividend news. The Basu et al. (2006) productivity measure, in contrast, produces an exposure coefficient not statistically different from zero.

The second panel of Table 1 reports the estimates of the ARMA(1,1) model. Across all productivity measures, the amount of predictability as measured by the $R^2$ increases substantially compared to the AR(1) case. Furthermore, for both the BLS and the Basu et al. (2006) productivity measures, we reject the restriction that the MA root is null, implying that the ARMA (1,1) process cannot be rejected in favor of the parsimonious AR(1) process. Most importantly, the estimated persistence of

[4] I use monthly CRSP value-weighted returns to construct the price-dividend ratio. Dividends are time-aggregated to an annual frequency, and prices are measured at the end of the year.

the long-run component is now close to one, consistent with the Bansal and Yaron (2004) findings for consumption. The results on the correlation between long-run productivity and price-dividend news are consistent with those found under the AR model.

Turning our attention to the bivariate approach, the first thing to notice is that the identifying parameter $\sigma_x$ is always precisely estimated and statistically different from zero. This result confirms the existence of predictability in productivity growth and enables all the other parameters to be identified. The long-run shocks are small and have a standard deviation ranging from 5% to 32% of that of the short run-shocks. Consistent with the ARMA(1,1) results, the long-run component persistence is very close to one. In contrast to the ARMA(1,1) specification, however, long-run and short-run shocks have a contemporaneous correlation, $\rho_{ax}$, which is basically zero. Finally, note that the sizable estimates of $\alpha_2$ confirm that asset prices substantially adjust to long-run productivity news, as predicted by the long-run risk paradigm.

The last panel of Table 1 reports the key parameters of the long-run component estimated using the BKY approach. This method confirms all the results obtained through the bivariate approach, and it predicts more sizeable long-run shocks. The smallest point estimate of the standard deviation of the long-run shocks is now 10% of that of the short-run shocks.

Fig. 1 shows a subset of the fitted time-series obtained using the bivariate (left panel) and the multivariate (right panel) approach. The left panel plots the BLS and the Basu et al. (2006) productivity growth time-series along with their respective long-run components. The right panel compares the BLS productivity with the Solow residuals. In all cases, the conditional mean of productivity slowly declines in the 1970s, and then it begins to grow again until the 1990s. These findings are consistent with those observed for other macroeconomic quantities and support the plausibility of the results obtained so far. Furthermore, the pattern of the estimated long-run components is similar. This is a reassuring result: different ways of using information from asset prices do not dramatically alter the basic time-series properties of the productivity long-run component.

### 2.1. *Macroeconomic significance*

What is the role of short-run and long-run risk for macroeconomic variables such as output, investment, consumption, labor, and Tobin's Q? Is the long-run risk component relevant only for asset prices, or is it connected to macroeconomic fundamentals? To answer these questions, this section investigates the explanatory power of both short-run and long-run risk for the growth rates of the aforementioned variables.

The main results are reported in Table 2 and are based on the BKY estimation procedure, as it represents a common benchmark in the long-run risk literature. The bivariate approach delivers overall similar results, which are omitted for brevity. For both the Solow residuals and the BLS productivity measure, the quantities of interest are projected on contemporaneous productivity shocks. Basu et al. (2006) note that aggregate quantities tend to respond with a delay to their estimated productivity news. For this reason, when employing the Basu et al. (2006) measure I project the variables of interest on one-year-lagged news.

The model presented in the next section produces a high equity premium because it assumes that positive long-run productivity news produces (i) a sharp drop in the stochastic discount factor (as in the endowment economy of Bansal and

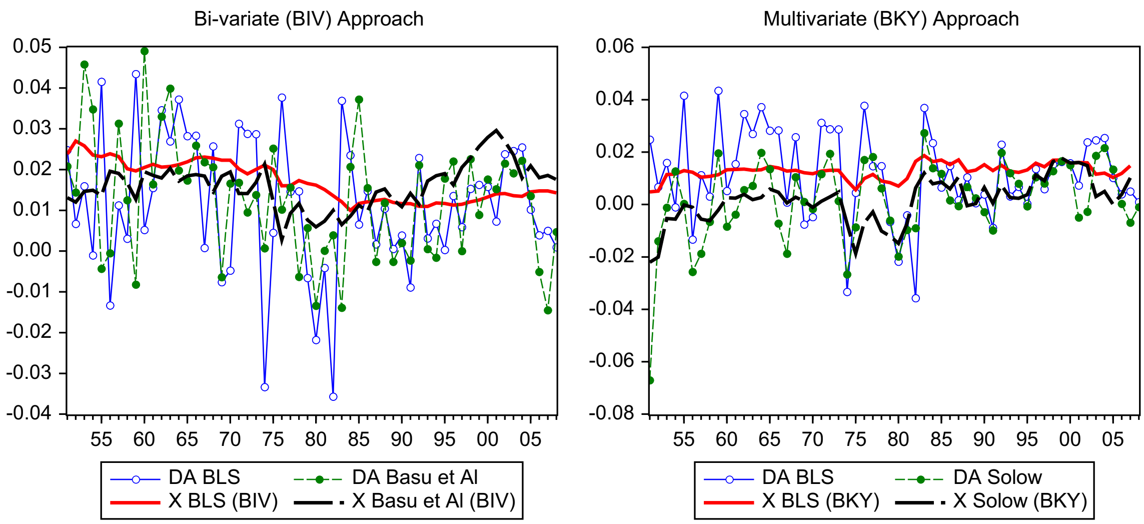


**Fig. 1.** Long-run productivity risk. *DA* denotes productivity growth rate; *X* refers to the fitted long-run component. In the left [right] panel, the long- run components are obtained according to the bivariate (BIV) [multivariate (BKY)] approach. The left panel compares results across the BLS and the Basu et al. productivity measures. The right panel compares results across the BLS and the Solow residuals productivity measures over their common sample. All parameter estimates are reported in Table 1.

**Table 2**
Macroeconomic growth in the long- and short-run.

| Productivity | Solow ($j=0$) | BLS ($j=0$) | Basu et al. ($j=1$) |
|---|---|---|---|
| $\Delta c_{t+1+j} = \mu_c + \beta_{\Delta c\|x} x_t + \beta_{\Delta c\|\epsilon_a} \epsilon_{a,t+1} + \beta_{\Delta c\|\epsilon_x} \epsilon_{x,t+1}$ | | | |
| $\beta_{\Delta c\|x}$ | 0.579** | 2.385*** | 5.736*** |
| $\beta_{\Delta c\|\epsilon_a}$ | 0.344*** | 0.263*** | 0.088** |
| $\beta_{\Delta c\|\epsilon_x}$ | −0.255* | −1.632** | 1.673*** |
| $R^2$ | 0.27 | 0.35 | 0.2 |
| $\Delta i_{t+1+j} = \mu_i + \beta_{\Delta i\|x} x_t + \beta_{\Delta i\|\epsilon_a} \epsilon_{a,t+1} + \beta_{\Delta i\|\epsilon_x} \epsilon_{x,t+1}$ | | | |
| $\beta_{\Delta i\|x}$ | 1.426*** | 3.862* | −4.121 |
| $\beta_{\Delta i\|\epsilon_a}$ | 2.801*** | 2.154*** | 0.724*** |
| $\beta_{\Delta i\|\epsilon_x}$ | 0.092 | 0.718 | 0.658 |
| $R^2$ | 0.41 | 0.38 | 0.04 |
| $iy_{t+1+j} = \mu_{iy} + \beta_{-1} iy_{t+j} + \beta_{iy\|\epsilon_a} \epsilon_{a,t+1} + \beta_{iy\|\epsilon_x} \epsilon_{x,t+1}$ | | | |
| $\beta_{iy\|\epsilon_a}$ | 0.337*** | 0.303*** | 0.095*** |
| $\beta_{iy\|\epsilon_x}$ | 0.304*** | 0.421 | 0.171* |
| $R^2$ | 0.92 | 0.71 | 0.65 |
| $q_{t+1+j} = \mu_q + \beta_{-1} q_{t+j} + \beta_{q\|\epsilon_a} \epsilon_{a,t+1} + \beta_{q\|\epsilon_x} \epsilon_{x,t+1}$ | | | |
| $\beta_{q\|\epsilon_a}$ | 1.744*** | 0.296 | 2.452*** |
| $\beta_{q\|\epsilon_x}$ | 1.014 | 8.009*** | 3.581** |
| $R^2$ | 0.81 | 0.86 | 0.79 |
| $\Delta y_{t+1+j} = \mu_y + \beta_{\Delta y\|x} x_t + \beta_{\Delta y\|\epsilon_a} \epsilon_{a,t+1} + \beta_{\Delta y\|\epsilon_x} \epsilon_{x,t+1}$ | | | |
| $\beta_{\Delta y\|x}$ | 0.778* | 2.735** | 5.365*** |
| $\beta_{\Delta y\|\epsilon_a}$ | 0.881*** | 0.681*** | 0.215*** |
| $\beta_{\Delta y\|\epsilon_x}$ | −0.197 | −1.139 | 1.314 |
| $R^2$ | 0.41 | 0.43 | 0.08 |
| $\Delta n_{t+1+j} = \mu_n + \beta_{\Delta n\|x} x_t + \beta_{\Delta n\|\epsilon_a} \epsilon_{a,t+1} + \beta_{\Delta n\|\epsilon_x} \epsilon_{x,t+1}$ | | | |
| $\beta_{\Delta n\|x}$ | 0.024 | 1.578* | 5.690*** |
| $\beta_{\Delta n\|\epsilon_a}$ | −0.113 | 0.433*** | −0.053 |
| $\beta_{\Delta n\|\epsilon_x}$ | −0.443 | −3.194** | 2.721* |
| $R^2$ | 0.026 | 0.38 | 0.08 |

This table shows the link between several aggregate variables and short-run shocks, $\epsilon_a$, long-run news, $\epsilon_x$, and long-run productivity $x$. The variables $\epsilon_{a,t}$, $x_t$, and $\epsilon_{x,t}$ have been computed using the point estimates reported in Table 1 for the BKY procedure. Data are annual (1950–2008). The symbol $\Delta$ refers to a log growth rate. Consumption, investment, output, labor, Tobin's Q, and the investment-output ratio are denoted as $c$, $i$, $y$, $n$, $q$, and $iy$, respectively.

* I denote $p$-values smaller than 10%, respectively.
** I denote $p$-values smaller than 5%, respectively.
*** I denote $p$-values smaller than 1%, respectively.

Yaron, 2004), and (ii) a significant rise in Tobin's Q due to convex adjustment costs and higher investment. One of the goals of Table 2 is to provide guidance on the plausibility of these channels.

Across all productivity measures, the long-run component in productivity has a significantly positive impact, $\beta_{\Delta c|x}$, on consumption growth (first panel, Table 2).[5] In the model, this channel is key for the generation of sizeable fluctuations in the stochastic discount factor. Turning our attention to contemporaneous responses, we see that across all productivity measures consumption growth is positively exposed to short-run shocks. This result is consistent with both previous empirical investigations and the implications of my benchmark model.

The immediate response of consumption to long-run news, $\beta_{\Delta c|\epsilon_x}$, is negative under both the BLS and the Solow residual productivity measures, whereas it is positive under the Basu et al. (2006) productivity measure (as in Barsky and Sims, 2011). These mixed results are not surprising in the context of the long-run risk literature: in small samples, identifying precisely contemporaneous responses is rather difficult. From a pure asset pricing perspective, whether $\beta_{\Delta c|\epsilon_x}$ is positive or negative is not extremely relevant in a long-run risk model, since the short-run response of consumption growth has a small market price of risk. In Bansal and Yaron (2004), for example, this response is assumed to be null. In my benchmark model, this response is negative and it tends to partially offset the adjustment of the stochastic discount factor, implying that obtaining a high equity premium becomes slightly more difficult.

[5] The magnitudes of the coefficients are not directly comparable across columns because the fitted long-run risk components have different volatilities.

Both the second and the third panels of Table 2 focus on the behavior of investment. The second panel shows that (i) long-run productivity growth produces long-run investment growth, and (ii) investment growth is positively exposed to short-run productivity news. The point estimate of the exposure to long-run shocks, $\beta_{\Delta i|\epsilon_x}$, is positive across all productivity measures, but statistically not different from zero. In order to improve the goodness of fit of my regressions and sharpen my inference, I also focus on the response of the investment-to-output ratio to both short- and long-run productivity news. Under this regression specification, the immediate response of investment to long-run shocks is always positive, and it is statistically significant under both the Solow residual and the Basu et al. (2006) productivity measures. Furthermore, as shown in the fourth panel, there exists a positive link between Tobin's Q, investment, and productivity news. This suggests that using simple convex adjustment costs could be a reasonable way to capture the link between asset prices and macroeconomic aggregates in the data.

The last two panels of Table 2 examine the effect of both short- and long-run productivity news on output and labor growth. The data suggest that long-run productivity risk also has a significant and positive impact on the expected growth rate of labor and output ($\beta_{\Delta y|x} > 0$ and $\beta_{\Delta n|x} > 0$). The contemporaneous response of output to short-run productivity news is always positive and is statistically significant. In contrast, the contemporaneous response of output to long-run news is never well identified. This is likely explained by the fact that the response of labor to long-run shocks, $\beta_{\Delta n|\epsilon_x}$, is marginally significant in two cases out of three. Also, the estimate of $\beta_{\Delta n|\epsilon_x}$ has opposite signs depending on whether productivity is adjusted by utilization. Although relevant, addressing the contemporaneous response of labor growth to productivity news goes beyond the scope of this project. For this reason, my theoretical analysis focuses mainly on the implications of the model for investment and equity returns.

In summary, Table 2 suggests the existence of a common productivity-based long-run component driving all major macroeconomic aggregates. The existence of predictability in productivity growth and its implications for consumption, investment, and output motivate and support the model explored in the next sections.

## 3. Model and main results

The representative agent has Epstein and Zin (1989) preferences defined over the consumption bundle $\tilde{C}_t$:

$$U_t = [(1-\delta)\tilde{C}_t^{1-1/\Psi} + \delta(E_t[U_{t+1}^{1-\gamma}])^{(1-1/\Psi)/(1-\gamma)}]^{1/(1-1/\Psi)}.$$

The parameters $\Psi$ and $\gamma$ measure the IES and the RRA of the agent, respectively. The consumption bundle aggregates consumption, $C_t$, and leisure, $l_t$, as follows:

$$\tilde{C}_t = [oC_t^{1-1/\xi_l} + (1-o)(A_{t-1}l_t)^{1-1/\xi_l}]^{1/(1-1/\xi_l)},$$

where $A$ denotes aggregate productivity and $o$ is a weight determining the average share of hours worked. Multiplying leisure by productivity guarantees balanced growth even when $\xi_l \neq 1$, and it is interpreted as an adjustment for the standards of living.

Output is produced according to a constant returns-to-scale neoclassical production function:

$$Y_t = K_t^{\alpha}[A_t n_t]^{1-\alpha},$$

where $K_t$ is the fixed stock of capital carried into date $t$, and $n_t$ is the labor input at $t$. The productivity growth rate, $\Delta a_{t+1} \equiv \log(A_{t+1}/A_t)$, has a long-run risk component and evolves as in Eq. (1).

The other constraints in this economy are as follows:

$$\begin{aligned} &Y_t \geq C_t + I_t, \qquad 1 \geq n_t + l_t, \\ &K_{t+1} \leq (1-\delta_k)K_t + I_t - G_t K_t, \end{aligned} \tag{4}$$

where $\delta_k$ denotes depreciation, $I_t$ is investment, and $G_t$ allows for convex adjustment costs. As in Jermann (1998), the cost function satisfies the restrictions $\overline{G} = \overline{G'} = 0$ at the steady state, and it is specified as follows:

$$G_t = \frac{I_t}{K_t} - \left[\frac{\alpha_1}{1-\frac{1}{\xi}}\left(\frac{I_t}{K_t}\right)^{1-1/\xi} + \alpha_0\right]. \tag{5}$$

In this economy, it is possible to find the competitive equilibrium allocation by solving the planner's problem. Once the planner's allocation is found, prices and returns can be derived as follows. The stochastic discount factor takes the following usual form:

$$M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-1/\xi_l}\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{1/\xi_l - 1/\Psi}\left(\frac{U_{t+1}}{E_t[U_{t+1}^{1-\gamma}]^{1/(1-\gamma)}}\right)^{1/\Psi-\gamma}.$$

The last factor relates to news regarding the continuation value of the representative agent. Future utility is very sensitive to long-run news, and for this reason it can induce high volatility in the stochastic discount factor even for moderate amounts of risk aversion.

The risk-free rate is $R_t^f = E_t[M_{t+1}]^{-1}$, and the unlevered equity returns are

$$R_{t+1} \equiv \frac{\alpha \frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta_k) + Q_{t+1}\left[G'_{t+1}\frac{I_{t+1}}{K_{t+1}} - G_{t+1}\right]}{Q_t}, \tag{6}$$

where $Q_t$ is equal to the marginal rate of transformation between new capital and consumption:

$$Q_t = \frac{1}{1-G'_t}. \tag{7}$$

The optimality condition for labor implies that marginal rate of substitution between consumption and leisure should equal the marginal product of labor:

$$\frac{\partial \tilde{C}_t}{\partial l_t} \Big/ \frac{\partial \tilde{C}_t}{\partial C_t} = (1-\alpha)\frac{Y_t}{n_t}. \tag{8}$$

This benchmark version of the model—as well as all other versions discussed in this paper—is solved through the perturbation methods provided in the dynare++ package.

### 3.1. Benchmark calibration

As in Bansal and Yaron (2004), I calibrate the model to a monthly frequency and then focus on time-aggregated statistics matching the behavior of US macroeconomic variables of interest over the long sample 1929–2008. Section 4 also considers a quarterly calibration. The monthly parameters are summarized in Panel A of Table 3.

The parameter $\mu$ is set to yield an annual average growth of 1.8% and $\sigma = 1\%$ to obtain an annual volatility of output growth of 3.34% under my benchmark calibration. This value for $\sigma$ is conservative, as the volatility of US output growth in the long sample 1929–2008 is 3.56%. The long-run component in productivity is calibrated so as to be relatively small but persistent, as seen in the previous section. The parameter $\rho$ is set to a conservative and empirically plausible number that in annualized terms implies a persistence of 0.80. To keep the long-run component as small as in the data, $\sigma_x = 10\% \cdot \sigma$ must be imposed. This number falls in the middle of the estimates range shown in Table 1 for $\sigma_x/\sigma$.

On the technology side, the annualized capital depreciation rate is 6%. The parameter $\alpha$ is calibrated to match the capital income share. The elasticity of the adjustment cost function, $\xi$, is set to 7, so that investment can be sufficiently volatile.

**Table 3**
Predictions for quantities and prices.

Panel A: Benchmark calibration

| $12\cdot\mu$ | $\sigma\sqrt{12}$ | $\sigma_x/\sigma$ | $\rho 12$ | $12\cdot\delta_k$ | $\alpha$ | $\xi$ | $\xi_l$ | $\delta^{12}$ | $\gamma$ | $\psi$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.8% | 3.35% | 0.10 | 0.80 | 0.06 | 0.34 | 7 | 1 | 0.95 | 10 | 2 |

Panel B: Main moments

| | DATA | IES > 1 ($\psi=2$) | IES < 1 ($\psi=0.9$) | No LRR ($\sigma_x=0$) | No costs ($\xi=0$) | BCF Friction | Reallocation Cost |
|---|---|---|---|---|---|---|---|
| $\sigma(\Delta y)$ (%) | 03.56 (00.65) | 03.34 | 03.38 | 02.33 | 03.87 | 03.66 | 03.37 |
| $\sigma(\Delta c)/\sigma(\Delta y)$ | 00.71 (00.05) | 00.81 | 00.75 | 00.64 | 00.54 | 00.64 | 00.93 |
| $\sigma(\Delta i)/\sigma(\Delta y)$ | 04.49 (00.61) | 03.61 | 06.40 | 02.39 | 06.77 | 02.82 | 03.25 |
| $E[I/Y](\%)$ | 20.00 (00.97) | 31.11 | 30.81 | 23.35 | 27.53 | 27.60 | 35.37 |
| $\rho(\Delta c, \Delta i)$ | 00.39 (00.28) | 00.35 | −00.02 | 00.85 | 00.29 | 00.64 | 00.52 |
| $\rho(\Delta c, r_{ex}^{LEV})$ | 00.25 (00.12) | 00.06 | −00.11 | 00.18 | 00.00 | 00.14 | 00.18 |
| $E[r_{ex,t+1}^{LEV}]$ (%) | 04.71 (02.25) | 05.25 | −05.12 | 01.48 | 01.11 | 01.93 | 04.07 |
| $\sigma(r_{ex,t+1}^{LEV})$ (%) | 20.89 (02.21) | 12.47 | 13.60 | 11.68 | 11.15 | 11.66 | 14.45 |
| $\sigma(q)$ | 00.29 (00.05) | 00.06 | 21.93 | 00.01 | 00.00 | 00.01 | 00.10 |
| $E[r_t^f]$ (%) | 00.65(00.38) | 00.94 | 21.27 | 05.07 | 04.92 | 00.53 | 00.94 |
| $\sigma(r_t^f)$ (%) | 01.86 (00.32) | 00.94 | 23.59 | 00.60 | 00.23 | 01.77 | 01.23 |
| $ACF_1[r_{ex,t+1}^{LEV}]$ | 00.09 (00.12) | 00.00 | 00.01 | −00.01 | −0.03 | 0.00 | −00.01 |
| $ACF_1[r_t^f]$ | 00.64 (00.06) | 00.35 | 00.94 | 00.00 | 00.98 | 00.23 | 00.32 |
| $ACF_1[q]$ | 00.86 (00.08) | 00.91 | 00.83 | 00.91 | 00.33 | 00.00 | 00.98 |
| $ACF_1[\Delta c]$ | 00.50 (00.15) | 00.59 | 00.52 | 00.30 | 00.71 | 00.71 | 00.59 |

Panel A shows the benchmark monthly calibration. The parameter $o$ is calibrated to deliver $n$=0.18 at the steady state. Under the benchmark calibration, $o$=0.205. The parameters $\alpha_1$ and $\alpha_0$ in the Jermann's adjustment cost function are determined by the following two restrictions: $\overline{G}=0$ and $\overline{G}'=0$. For the reallocation costs defined in Eq. (14), I set $a_1 = 9.e^{-6}$ and $\tau = 1.5$; the smooth approximation of the absolute value features $\epsilon = 100$. *BCF* refers to Boldrin et al. (2001). In panel B, all the statistics are expressed in annual terms. Data are annual (1929–2008); numbers in parentheses are Newey–West adjusted standard errors obtained through a GMM estimator. The entries for the models are based on 100 simulations each with 840 monthly observations that are time-aggregated. Small letters denote log-units. E[·], $\sigma(\cdot)$, $\rho(\cdot,\cdot)$, and $ACF_1[\cdot]$ denote mean, volatility, correlation, and first-order autocorrelation, respectively.

On the preference side, the consumption bundle elasticity $\xi_l$ is set to one. Sensitivity analysis with respect to this parameter is provided in Section 4. The parameter $o$ is calibrated so that the share of hours worked is 18% at the steady state, as in Tallarini (2000). The RRA and the IES are set to values of 10 and 2, respectively. These values are consistent with the estimates of Attanasio and Vissing-Jorgensen (2003); Bansal et al. (2007); Bansal et al. (2010); and Colacito and Croce (2011). The annualized subjective discount factor is fixed at 0.95.[6]

In the data, excess returns are levered and a substantial part of the financial dividend growth volatility is due to idiosyncratic payout shocks. To better compare the results of the model to the data, I also look at the following excess returns:

$$R_{ex,t}^{LEV} = \phi_{lev}(R_t - R_{t-1}^f) + \epsilon_t^d, \tag{9}$$

where $\phi_{lev} = 2$, and $\epsilon_t^d \sim i.i.d.N(0, \sigma_d)$. My calibration of $\phi_{lev}$ is consistent with the amount of financial leverage measured by Rauh and Sufi (2012) and is conservative with respect to the empirical findings of Garcia-Feijo and Jorgensen (2010). The cash-flow shock, $\epsilon_t^d$, is not priced and hence does not alter the equity premium. This shock only affects the volatility of the excess returns, and its annualized volatility, $\sigma_d\sqrt{12}$, is set to 6.5%, consistent with Bansal and Yaron (2004). Without this shock, the equity Sharpe ratio becomes excessively large.

### 3.2. Predictions on quantities and prices

The main results produced by my benchmark calibration are reported in panel B of Table 3 in the column IES > 1. The model reproduces the relative variance of consumption to output and generates an amount of investment volatility that falls within the confidence interval in the data. The average investment-output ratio is 30%, a number consistent with post-war data but higher than its empirical counterpart in the pre-World War II sample.

To better judge this result, note that in any standard RBC model with decreasing returns to capital, the average investment-output ratio is a negative function of the average unlevered capital return, $E[R_t]$. In order to obtain a low investment-output ratio, the model has to produce either a high risk-free rate or a high risk premium. Models with standard time-additive CRRA preferences produce basically a null risk premium and match the low level of the investment-output ratio only by targeting a risk-free rate four to six times higher than in the data. In my benchmark model, however, this tension is almost completely resolved thanks to the risk premium channel. Specifically, the unconditional average of the risk-free rate is 0.94%, whereas the average unlevered annual capital return is 3.6% because of the presence of a high risk premium. The implied levered equity premium of the model is 5.25%, slightly higher than the historical mean of the CRSP stock market log excess returns. This result represents an accomplishment relative to prior literature, especially considering that the relative risk aversion is set to 10, a moderate level.

To understand the reasons that the model is able to generate such a high equity premium, note that the Euler equation implies that $E_t[r_{t+1} - r_{f,t}] \approx -cov_t(m_{t+1}, r_{t+1})$. Capital must offer a higher equity premium when its returns move in a direction opposite to that of the discount factor. In Fig. 2, I show the impulse response function of both quantities and asset prices after short- and long-run shocks in order to better highlight their co-movements. The shocks materialize only at time 1, and they are normalized according to their monthly standard deviations ($\epsilon_{a,1} = \sigma$ and $\epsilon_{x,1} = \sigma_x$).

When a short-run shock materializes, the representative agent finds it optimal to increase consumption and investment at the same time. Increasing investment allows the representative agent to temporarily increase the capital stock and smooth consumption over time. This impulse response is fully consistent with the predictions of any standard RBC model.

The behavior of consumption and investment is very different with respect to the long-run component. First of all, notice that long-run shocks are very long-lasting, and for this reason they can have a very strong impact on savings decisions even if the shocks are small. Furthermore, long-run news simultaneously generates both an income effect and a substitution effect, which work in opposite directions. Higher expected long-run productivity generates a substitution effect that increases the opportunity cost of consumption, i.e., it tends to stimulate investment and reduce consumption. At the same time, a positive long-run shock allows the agent to feel much richer and to desire an immediate increase in consumption. The long-run component is highly persistent, and thus a single long-run shock is able to affect the flow of expected future utility over a very long time-horizon. Indeed, a positive long-run shock translates into a remarkable increase in the continuation value of the agent. For a given output, the income effect tends to produce an increase in consumption and a drop in investment.

When the IES is sufficiently high, as in the benchmark calibration, the degree of substitutability between continuation value and current consumption is also high. In this case, the substitution effect dominates the income effect, and the agent finds it optimal to invest more in order to accumulate more capital. This is the reason that after a positive long-run shock, consumption growth drops while investment growth increases. Because of complementarity between consumption and

[6] When $\xi_l = 1$, my consumption bundle is $\tilde{C}_t = C_t^o (A_{t-1} l_t)^{1-o}$, whereas Tallarini (2000) adopts the bundle $\tilde{C}_t = C_t l_t^{(1-o)/o}$. Both bundles preserve homogeneity of degree one of the utility function with respect to productivity and deliver the same utility dynamics for a wide range of IES values. Under the benchmark calibration with IES = 2, the Tallarini aggregator produces a counterfactual contraction of labor with respect to positive short-run shocks. This problem can be resolved, for example, by slightly lowering the IES to 1.8 and imposing $\rho^{12} = 0.81$.

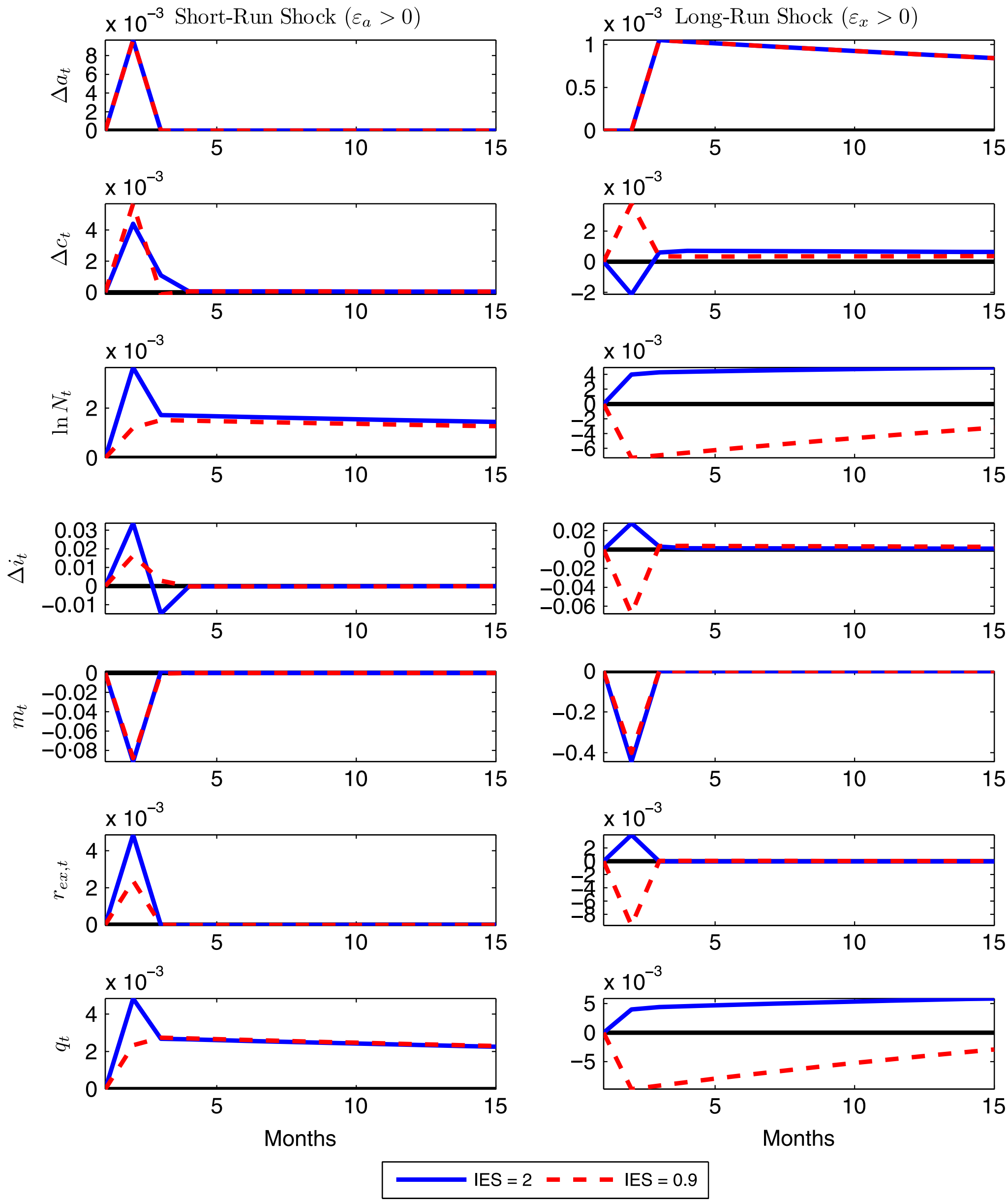


**Fig. 2.** The role of IES ($\Psi$). This figure shows monthly log-deviations from the steady state. Excess returns are not levered. All the parameters are calibrated to the values reported in Table 3.

leisure, the agent also finds it optimal to reduce leisure. As a result, both labor and total output increase to better support the investment boom.

Under the benchmark calibration, investment rises after both positive short- and long-run shocks, consistent with the empirical evidence in Section 2. This creates a pressure on the price of capital to appreciate, so that both short-run and long-run shocks imply bigger stock market returns and a contemporaneous fall in the stochastic discount factor. Stock market returns are indeed risky with respect to both sources of uncertainty, and for this reason they command the high equity premium reported in Table 3, despite their low correlation with short-run consumption growth.

The excess returns implied by the model are unfortunately less volatile than those observed in the data. This is due to the fact that Tobin's Q in the model is not volatile enough. Overall, however, thanks to the presence of adjustment costs, the volatility is much higher than that observed in standard frictionless production-based models (Tallarini, 2000; Kaltenbrunner and Lochstoer, 2010, and the CRRA case in Jermann, 1998). My economy also reproduces the empirical autocorrelation of both the price-dividend ratio and the excess returns.

The risk-free rate is persistent and has an annual standard deviation of just 0.94%. This result is a success with respect to standard habit models, which are widely known to produce an excessively volatile risk-free rate (see, for example, Jermann, 1998; Boldrin et al., 2001).

Co-movements are an important dimension that this model is also able to explore. The contemporaneous correlation of unfiltered consumption and investment is moderate in the data. Indeed, while at business cycle frequencies consumption and investment are highly correlated, at lower frequencies they are not. The model is perfectly able to reproduce this moderate correlation thanks to the presence of the long-run component. As seen before, while short-run shocks induce a perfect correlation between consumption and investment growth, long-run news forces consumption and investment to move in opposite directions. Overall, the correlation between these two variables implied by the model is still positive and perfectly in line with the empirical evidence.

The excess returns are poorly correlated with consumption growth: in the data this correlation is about 21%—basically null—while in the model it is 6% for the levered excess returns and 25% for the unlevered excess returns. This result is extremely significant because it shows that the model produces high a equity premium without altering the total correlation between short-run consumption growth and stock market returns. This is one of the novelties of this type of model.

### 3.3. The role of IES and long-run risk

This section examines a deviation from my benchmark calibration in order to highlight the role of the IES and the long-run risk component.

*Investment dynamics and IES*: The IES is a key parameter in this economy and in all the long-run risk models in general. Bansal and Yaron (2004) show that an IES greater than one is important to both maintain the unconditional mean of the interest rate at a low level and replicate the predictability of the excess returns in the presence of time-varying aggregate volatility. Under my benchmark calibration, I abstract away from the presence of stochastic volatility, and for this reason my excess returns do not display any significant level of predictability. Still, the IES plays a crucial role in determining the response of endogenous quantities and, hence, the unconditional mean of the excess returns.

To better demonstrate this point, the model is also solved for the case in which the IES is 0.9 and all other parameters are unchanged. The ultimate goal is to show that the results can change substantially even when the IES is slightly below unity. Considering lower values of the IES would amplify even further my current findings.

The impulse responses related to the case of IES$=$0.9 are plotted in Fig. 2 using dashed lines. No matter whether the IES is set to 2 or 0.9, the agent finds it optimal to respond to positive short-run shocks by increasing consumption, labor, and investment simultaneously. This is the reason that all the responses in the left panels are qualitatively the same.

The behavior of consumption and investment differs, however, with respect to the long-run component. As mentioned before, this is due to the interaction between the IES, the income effect, and the substitution effect produced by long-run news. When a positive shock to long-run productivity materializes, we know that the continuation value increases significantly. If the IES is low enough, current consumption and continuation value are complements: the income effect dominates, and the agent finds it optimal to increase consumption by reducing investment, in contrast to what is observed for high values of the IES.

To compare these two different calibrations on a quantitative level, the column IES $<1$ of Table 3 reports the results from the model with a low IES. When the IES$=$0.9 there are three problems: (i) the response of investment to long-run shocks becomes excessive and its contemporaneous correlation with consumption turns negative; (ii) the risk-free rate becomes too high and volatile; and (iii) the equity premium changes sign.

The origin of the negative risk premium becomes clear if we look once again at Fig. 2 and focus on the response of the market returns to long-run productivity shocks. If the IES is too low, the returns fall below steady state in good times. This occurs because investment demand drops, producing a substantial capital loss that is inconsistent with the results of Section 2. Under this calibration, the market functions as an insurance device with respect to long-run news, and for this reason it pays a much lower premium. In this framework, asset prices clearly suggest that one should adopt an IES higher than the one used in the standard RBC literature. The analysis conducted so far also shows that a higher IES produces good results for the co-movements of the macro variables.

*Short-run productivity risk only*: What do we gain from introducing long-run productivity risk into our macro models? Equivalently, what do we lose if we neglect long-run uncertainty? To answer these questions, Table 3 reports the model performance when productivity follows a random walk with constant drift (the NO LRR case). Neglecting long-run productivity uncertainty produces the following problems: (i) consumption becomes too smooth ($\sigma(\Delta c) = 1.50\%$ on an annual basis), and its contemporaneous correlation with investment becomes too high; (ii) the Tobin's Q fluctuations become basically negligible, and the equity premium goes from 5.25% under the benchmark calibration to just 1.48%; (iii) the risk-free rate is five times higher than before, and its volatility becomes much lower.

### 3.4. Capital accumulation frictions

This section explores the relevance of capital accumulation frictions in the context of a long-run risk–based production economy.

*No capital accumulation friction*: I begin by comparing the results obtained under the benchmark calibration with those produced by an economy in which there are no adjustment costs, i.e., $\xi \to +\infty$. The moments produced by this model are reported in Table 3 in the No Cost column; the relevant impulse response functions are reported in Fig. 3(a).

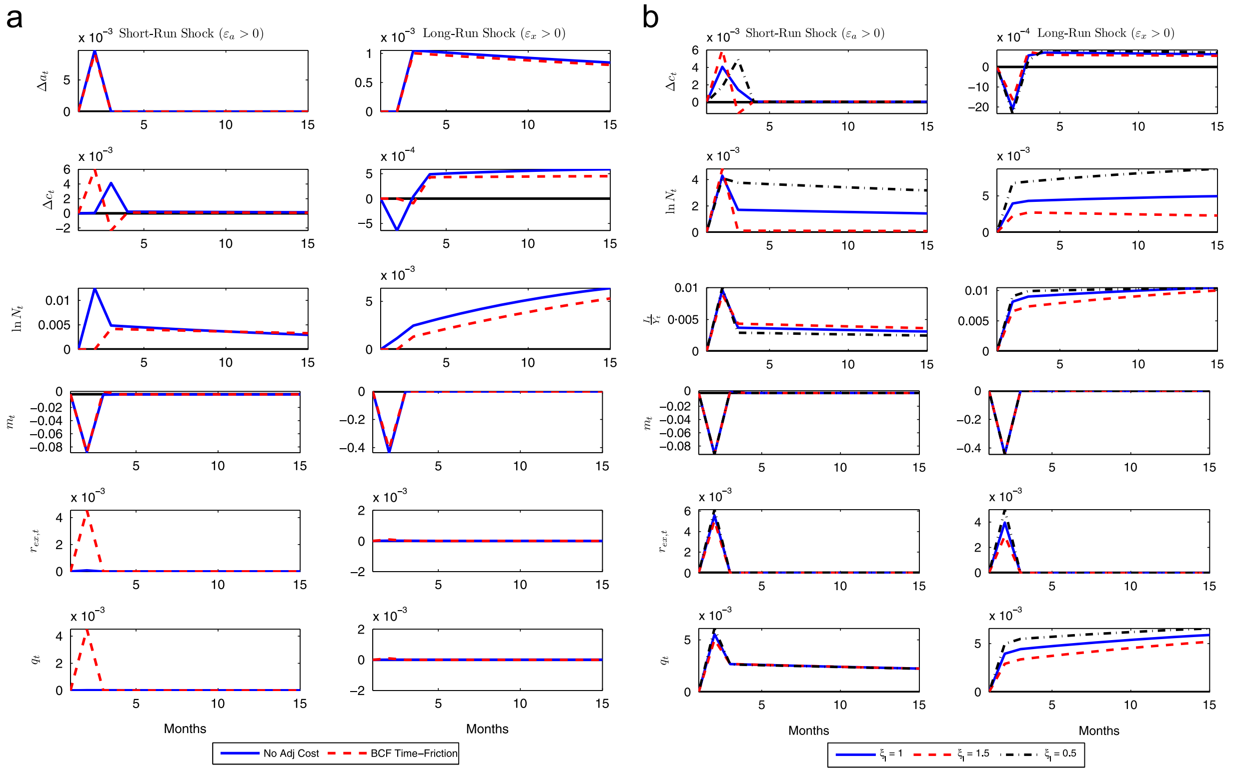


**Fig. 3.** Impulse response functions. (a) The role of the BCF time-friction and (b) the role of labor elasticity. This figure shows monthly log-deviations from the steady state. Excess returns are not levered. All parameters are calibrated to the values reported in Table 3. In panel (b), the elasticity of substitution between leisure and consumption is allowed to vary.

When the adjustment costs are not present, the agent finds it more convenient to intensively and persistently vary capital subsequent to both short-run and long-run shocks. As documented in Table 3, the absence of any investment friction produces the following problems: (i) investment becomes too volatile and less correlated with consumption growth because of the dominance of the long-run shock; (ii) the equity premium goes down to 1.1%, as Tobin's Q cannot vary and no significant excess return is realized upon the arrival of long-run news; and (iii) the risk-free rate is too high.

The interaction between the IES and the elasticity of the adjustment costs function is crucial to an understanding of the lower co-movement between consumption and investment. When the supply of new capital is flat and the IES is sufficiently high, the substitution effect dominates the income effect also in the short run. When the adjustment of capital is costless, the opportunity cost of consumption increases significantly, even after short-run shocks. If the agent has a high elasticity of substitution, she will find it optimal to leave consumption almost constant in order to invest more after the realization of a transitory positive shock to productivity growth.

Boldrin et al. (1997) *friction*: Boldrin et al. (1997) (henceforth BCF) suggest that Tobin's Q fluctuations could be fully generated by a time-friction in the allocation of inputs across sectors. Specifically, these authors show that if it takes one period to reallocate inputs across the investment and consumption sectors, excess returns could become substantially volatile and risky even in an RBC production economy without convex adjustment costs. My results show that in a long-run risk–based production economy, this time-friction is not able to produce results as powerful as the ones observed with Jermann (1998) adjustment costs.

To introduce the BCF time-friction into the benchmark model, the following three modifications are made. First, the parameter $\xi$ is set to $+\infty$ to remove convex adjustment costs. Second, I let $S_t \in [0,1]$ denote the share of output produced by the investment sector at time $t+1$ and introduce the following constraint:

$$I_t = S_{t-1} Y_t, \tag{10}$$

meaning that the share of output devoted to investment is one-period-ahead predetermined.[7] Eqs. (6)–(7) no longer hold, as the shadow value of capital is pinned down by the optimality condition with respect to $S_t$,

$$E_t[M_{t+1} Y_{t+1} Q_{t+1}] = E_t[M_{t+1} Y_{t+1}], \tag{11}$$

[7] Since in BCF the consumption and investment sectors feature the same constant return-to-scale production technology and the same productivity, allocating inputs across these two sectors is equivalent to simply deciding the investment-output share.

and the unlevered market return is now

$$R_{t+1} \equiv \frac{\alpha \frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta_k) + S_t(Q_{t+1}-1)\alpha \frac{Y_{t+1}}{K_{t+1}}}{Q_t}.$$

Third, the total share of hours worked, $S_{n,t}$, is decided one period in advance:

$$n_t = S_{n,t-1}, \tag{12}$$

and the optimality condition for labor in Eq. (8) is replaced by the following condition:

$$E_t\left[M_{t+1}(1-\alpha)\frac{Y_{t+1}}{S_{n,t}}\right] = E_t\left[M_{t+1}\frac{\partial \tilde{C}_{t+1}}{\partial l_{t+1}} \Big/ \frac{\partial \tilde{C}_{t+1}}{\partial C_{t+1}}\right]. \tag{13}$$

The moments produced by this model are reported in Table 3 in the BCF Friction column. The results look similar overall to those obtained under the frictionless economy, with two exceptions: (i) investment is much smoother than before and more correlated with consumption; and (ii) the equity premium almost doubles and reaches an annual level of 2%. To understand these differences across the frictionless and BCF-based configurations of the model, it is useful to turn our attention to the impulse response functions depicted in Fig. 3(a).

With respect to short-run shocks, we see that consumption growth spikes up immediately upon the realization of the positive news, whereas in the frictionless model there is virtually no adjustment. This response is explained by the fact that when the share of consumption-to-output is predetermined, consumption has to grow exactly as much as output. Because the same is true for investment, investment growth becomes much less volatile than under the frictionless model. Since the total supply of labor is pre-determined as well, in the very first period labor does not adjust, but it perfectly 'catches up' with the frictionless model after one period. Because of complementarity between leisure and consumption, the agent reduces consumption growth in the second period and replicates the consumption time-series observed under the frictionless model afterwards.

Upon the realization of positive long-run news, neither consumption nor labor moves, because the news has no immediate impact on productivity and both labor and consumption are predetermined. From the second period on, however, consumption and labor track with the time-path observed under the frictionless model. Given that consumption and investment growth rates do not adjust in opposite directions upon the realization of long-run shocks, their overall correlation becomes higher than in both the frictionless model and the data.

Turning our attention to the value of capital, $q_t$, we see that it strongly and positively responds to short-run news, as in Boldrin et al. (1997). Upon the realization of good but transitory productivity news, there is a strong incentive to increase investment in order to take immediate advantage of the temporary increase in productivity growth. Since the supply of investment goods is rigid, such a positive shock to the investment demand results in an increase of $q_t$. At the equilibrium, the implied positive capital excess return has to carry a higher equity premium.

Upon the realization of positive long-run shocks, in contrast, there is virtually no adjustment in the value of capital (the increase in $q_t$ is in the order of $5.e^{-5}$). The reason is that this shock is small on impact and very long-lasting, implying that a one-period delay in the investment path plays a very minor role in the determination of the future stream of capital income. As a result, the excess returns have almost null exposure to long-run news, as in the frictionless model.

*Convex reallocation costs*: Boldrin et al. (2001) assume that it takes one period to change the relative size of the investment and consumption sectors by reallocating inputs. As seen above, under this assumption asset prices are driven by the predetermined investment-output ratio, $S$. Here, the Boldrin et al. (2001) time-friction is turned into a convex reallocation friction by assuming that inputs can be reallocated from the consumption to the investment sector without any delay by paying the following cost:

$$G_t = [a_1 |S_t - \overline{S}|]^{\tau}, \tag{14}$$

where $S_t = I_t / Y_t$ is the current investment-output ratio and $\overline{S}$ is its steady-state level.

This setting retains all of the equations of my benchmark model except that Eq. (5) is replaced with (14). After this change, Eqs. (6)–(7) no longer hold and need to be replaced by the following two equations:

$$Q_t = \frac{1}{1 - G'_t \frac{K_t}{Y_t}}, \tag{15}$$

and

$$R_{t+1} \equiv \frac{\alpha \frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta_k) + Q_{t+1}\left[\alpha G'_{t+1} S_{t+1} - G_{t+1}\right]}{Q_t}. \tag{16}$$

Eq. (15) implies a positive relationship between Tobin's Q and the investment-output ratio, which is empirically plausible. As shown in Fig. 5(a), in the post-World War II period the contemporaneous correlation between the investment-output ratio and Tobin's Q has been positive and significant, consistent with both Eq. (15) and the empirical findings in Section 2.

I set $a_1 = 9.e^{-6}$ so that these costs are small (on average less than 0.5% of output), and $\tau = 1.5$ so that both the total volatility of output and the relative investment are comparable to those observed under the benchmark model with

Jermann's adjustment costs. To preserve the smoothness of all variables required to use perturbation methods, the Schmidt et al. (2007) smooth approximation of the absolute value,

$$|x| = \lim_{\epsilon \to +\infty} \frac{1}{\epsilon}\left[\ln\left(1+e^{-\epsilon x}\right)+\ln\left(1+e^{\epsilon x}\right)-2\,\ln(2)\right],$$

is applied to Eq. (14), with $\epsilon = 100$.

The moments obtained by employing convex reallocation costs are reported in the last column of Table 3. The main result is that convex reallocation costs perform very similarly to the Jermann's adjustment costs, implying that the long-run risk framework can produce a sizeable equity premium even in a two-sector economy designed in the spirit of Boldrin et al. (1997). As shown in Fig. 5(b), the dynamic response of all main variables of interest is very similar to that of the benchmark case with Jermann's costs, meaning that the intuitions developed under the benchmark model continue to hold.

## 4. Further inspection of the mechanism

This section perturbs my benchmark calibration in order to study the role of labor elasticity, different productivity specifications, the decision horizon, and time-varying volatility in productivity growth. The main results of this analysis are summarized in Table 4 and Fig. 5(c) and the main economic intuitions are addressed in what follows. For the sake of brevity, Table 4 reports only the moments that significantly change in the sensitivity analysis.

*Labor elasticity*: Panel A of Table 4 compares the benchmark calibration with Cobb-Douglas aggregation of leisure and consumption, $\xi_l = 1$, to the case in which $\xi_l$ is either greater or smaller than unity. First of all, note that changing $\xi_l$ alters only marginally the volatility of investment, the risk-free rate, the risk-premium and the contemporaneous correlation between consumption and investment growth.

To better understand these results, note that as the $\xi_l$ parameter increases, substitution between leisure and consumption is facilitated (left panels, Fig. 3(b)). As a result, the substitution effect induced by a positive short-run shock implies a stronger incentive to work and produce more. On the quantity side, higher output growth is associated with a more pronounced positive co-movement between consumption and investment growth. On the asset pricing side, we have a slightly less pronounced response of the excess return, meaning that the exposure of returns to short-run shocks becomes smaller and tends to reduce the equity premium.

The response of labor to long-run shocks, in contrast, becomes less pronounced as $\xi_l$ increases (right panel, Fig. 3(b)). As already discussed, when the IES is high enough, positive news shocks force consumption to drop in order to free up more resources for investment. When $\xi_l$ is low, a drop in consumption tends to facilitate a drop in leisure, i.e., the household has a stronger incentive to work. As the elasticity of substitution between consumption and leisure is increased, the agent finds it

**Table 4**
Inspecting the mechanism.

| Moments | Data | $\xi_l = 1$ | $\xi_l = 1.50$ | $\xi_l = 0.50$ |
|---|---|---|---|---|
| Panel A: The role of labor elasticity | | | | |
| $\sigma(\Delta i)/\sigma(\Delta y)$ | 04.49 (00.61) | 3.61 | 3.52 | 3.72 |
| $\rho(\Delta c, \Delta i)$ | 00.39 (00.28) | 0.35 | 0.51 | 0.19 |
| $E[r^{LEV}_{ex,t+1}]$ (%) | 04.71 (02.25) | 5.08 | 4.70 | 6.42 |
| $E[r^f_t]$ (%) | 00.65 (00.38) | 0.94 | 1.14 | 0.68 |
| $\beta_{\Delta c\mid\epsilon_a}$ | 00.34 (00.11) | 0.88 | 0.81 | 0.96 |
| $\beta_{\Delta c\mid\epsilon_x}$ | −00.26 (00.15) | −0.01 | −0.21 | 1.69 |
| $\beta_{\Delta iy\mid\epsilon_a}$ | 00.38 (00.01) | 0.70 | 0.77 | 0.72 |
| $\beta_{\Delta iy\mid\epsilon_x}$ | 00.22 (00.06) | −0.14 | 0.33 | −2.61 |
| $\beta_{q\mid\epsilon_a}$ | 01.50 (00.53) | 0.37 | 0.37 | 0.39 |
| $\beta_{q\mid\epsilon_x}$ | 02.23 (01.21) | 0.50 | 0.75 | −0.88 |
| | Data | Benchmark | AR(1) | ARMA(1,1) |
| Panel B: The role of different productivity specifications | | | | |
| $\sigma(\Delta i)/\sigma(\Delta y)$ | 04.49 (00.61) | 3.61 | 2.45 | 3.89 |
| $\rho(\Delta c, \Delta i)$ | 00.39 (00.28) | 0.35 | 0.80 | 0.60 |
| $E[r^{LEV}_{ex,t+1}]$ (%) | 04.71 (02.25) | 5.25 | 2.26 | 8.34 |
| $E[r^f_t]$ (%) | 00.65 (00.38) | 0.94 | 4.53 | 1.68 |
| $ACF_1[\Delta c]$ | 00.50 (00.15) | 0.59 | 0.49 | 0.83 |

All statistics are expressed in annual terms. Data are annual (1929–2008); numbers in parentheses are Newey–West adjusted standard errors obtained through a GMM estimator. The entries for the models are based on time-aggregated values obtained through repetitions of small-sample simulations. In panel A, all parameters are set to the values reported in Table 3 and $\xi_l$ is allowed to vary. In panel B, the benchmark calibration is altered by changing the productivity growth process. All calibration details are provided in Section 4. The first column in each panel reports the results from the benchmark model in Table 3, column IES > 1.

optimal to trade off more consumption goods for leisure in order to stabilize her consumption bundle. This explains why labor, output, and returns increase by less when $\epsilon_x > 0$ and $\xi_l > 1$. As a result, a higher elasticity of substitution (i) reduces the negative co-movement between investment and output coming from long-run shocks, and (ii) reduces the risk premium associated with long-run news. Note that across all these experiments, the subjective discount factor is unchanged, whereas the risk-free rate increases with $\xi_l$. Adjusting the subjective discount factor to keep the risk-free rate constant across different values of $\xi_l$ would make the changes in the equity premium negligible.

The elasticity of substitution between consumption and leisure plays a relevant role in the determination of the estimated responses of consumption, investment, and Tobin's Q to both short- and long-run shocks. The entries for the data column in Table 4 (from the first column of Table 2) are based on a Solow residual measure of productivity. This measure was chosen because in the context of the model I can time-aggregate quantities and compute annual Solow residuals, as in the data. In a second step, the simulated Solow residuals are projected on the model risk-free rate and price-dividend ratio to identify long- and short-run shocks and the regressions detailed in Table 2 are then run. This procedure yields results very comparable to the empirical ones, as it explicitly accounts for possible misestimations of the long-run shocks.

Under the benchmark calibration, the model captures well the sign, and often also the magnitude, of the exposure coefficients estimated in the data. In contrast to the data, however, the estimated exposure of the investment-output ratio to long-run news is negative. Using an elasticity greater than one, although not necessarily as high as 1.50, resolves this problem. Finally, note that even though the model underestimates the response of Tobin's Q to both short- and long-run shocks, it predicts values within the confidence intervals estimated in the data.

*Alternative productivity specifications*: Panel B of Table 4 compares the benchmark model with two models that differ only in their productivity process. Specifically, the column AR(1) refers to the results obtained when the growth rate of productivity evolves as an AR(1) with a persistence, $\rho$, of 0.50 to keep the autocorrelation of consumption aligned with the data. All other parameters and functional forms are set as in the benchmark model. This productivity process features only one source of shocks, and for this reason it fails to reproduce (i) the independent negative co-movement between consumption and investment originated by long-run news and (ii) the high volatility of investment relative to output. Furthermore, since $\rho$ is set at a lower rate, the overall equity premium drops from 5.25% to just 2.26%.

The last column of Panel B refers to the case in which productivity is assumed to evolve as an ARMA(1,1). In this case, $\rho$ is set as in the benchmark model and $b = 0.91\rho$ to replicate the same amount of relative volatility of the long-run component obtained under the benchmark calibration. The volatility of the ARMA(1,1) shock is adjusted to replicate the total volatility of productivity growth assumed under the benchmark model.

The details of these computations are reported in the appendix and are based on the results of Croce et al. (2007) because the ARMA(1,1) case corresponds to the steady-state solution of the Kalman filter problem adopted by an agent with limited information on the components of productivity growth. On the asset pricing side, it is important to note that this model produces a much higher equity premium because it restricts the long-run and short-run shocks to be perfectly correlated. This assumption, which is not supported by my empirical investigation, mechanically amplifies the market price of risk. On the quantity side, similarly to the AR(1) case, the model overestimates the correlation between consumption and investment growth.

*Decision horizon*: In the macroeconomic literature, models are often calibrated to a quarterly frequency to match the decision horizon in the model with business cycle frequency. In the long-run news literature, however, it is customary to use a monthly decision horizon, as financial investment decisions are taken at higher frequencies (Bansal and Yaron, 2004). As shown by BKY, the market price of risk associated with long-run shocks depends in a highly nonlinear and negative way on the decision horizon, i.e., the lower the frequency of the calibration of the model, the lower the implied risk premium.

This is exactly what happens when the model is recalibrated to a quarterly frequency and $\rho^4 = 0.80$ is imposed, in contrast to $\rho^{12} = 0.80$. The annual equity premium declines by 290 basis points, whereas all other moments remain basically unchanged. It is important to note that the model is still capable of producing an equity premium close to 5% under a less conservative calibration with $\rho^4 = 0.90$. The estimation results in Table 1 suggest that this calibration cannot be statistically rejected by the data.

*Time-varying volatility*. Several recent papers have documented a reduction in the volatility of aggregate variables (for example, Lettau et al., 2008; Fernandez-Villaverde and Rubio-Ramirez, 2006; Justiniano and Primiceri, 2008). This finding is very robust across all of my measures of productivity, as shown in Fig. 5(c) by means of simple GARCH(1,1) estimations.

To investigate the impact of time-varying volatility in the model, I consider the following specification for monthly productivity growth:

$$
\begin{aligned}
\Delta a_{t+1} &= \mu + x_t + e^{v_t}\sigma\epsilon_{a,t+1} \\
x_t &= \rho x_{t-1} + e^{v_t}\sigma_x\epsilon_{x,t} \\
v_t &= \rho_\sigma v_{t-1} + \sigma_\sigma\epsilon_{\sigma,t} \\
\begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \\ \epsilon_{\sigma,t+1} \end{bmatrix} &\sim iidN\left( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \rho_{v,x} \\ 0 & \rho_{v,x} & 1 \end{bmatrix} \right),
\end{aligned}
\tag{17}
$$

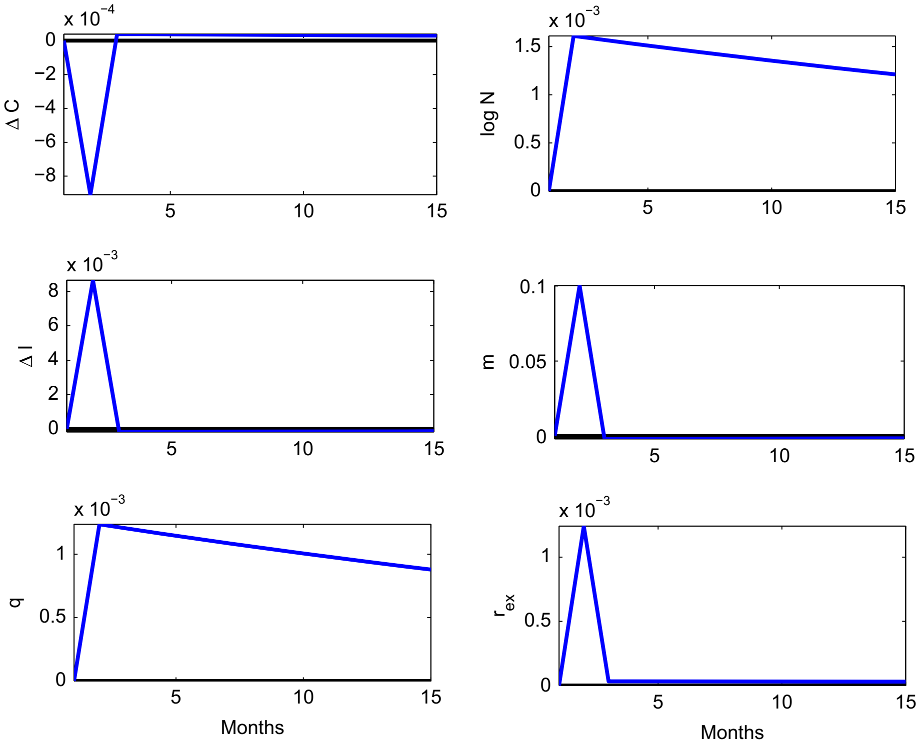


**Fig. 4.** Impulse responses with and without time-varying volatility. This figure shows monthly log-deviations from the steady state. All parameters are calibrated to the values reported in Table 3. Excess returns are not levered. The time-varying volatility process is specified in Eq. (17), with $\rho_\sigma = 0.987$, $\sigma_\sigma = 0.01$, and $\rho_{v,x} = 0$.

where the process $e^{v_t} > 0$ generates fluctuations in the total amount of uncertainty, in the spirit of Bansal and Yaron (2004). To make my log-volatility process comparable to the Bansal and Yaron (2004) volatility specification, $\rho_v = 0.987$ and $\sigma_v = 0.01$ are set so that StD.$[e^{v_t}]/E[e^{v_t}] = 7\%$, a number consistent with my annual estimations.[8] I then consider both the case in which volatility shocks are orthogonal to long-run shocks, $\rho_{v,x} = 0$, and the case in which they are countercyclical with respect to news, $\rho_{v,x} = -1$.

When volatility shocks are assumed to be orthogonal to long-run news shocks (Bansal and Yaron, 2004), the main departure from the benchmark model is in regard to the equity premium. In contrast to the predictions of the endowment economy of Bansal and Yaron (2004), the introduction of time-varying volatility reduces the equity premium, as independently pointed out by Malkhozov (2013).

To better see this point, Fig. 4 depicts the impulse responses of the model with time-varying risk with respect to a one-standard-deviation volatility shock. Upon the increase of uncertainty in the economy, the agent finds it optimal to save, work, and invest more in order to increase the stock of capital. Because of the increase in investment, the value of capital, $q$, increases as well and the stock market provides a positive excess return when the marginal utility of the agent is high, i.e., in a bad state of the world. The equity market hence provides insurance against orthogonal volatility shocks.

When $\rho_{v,x}$ is set to zero, the equity premium declines by 35 basis points. If, on the other hand, $\rho_{v,x}$ is set to $-1$, the equity premium remains basically unchanged. This is due to the fact that when a good long-run productivity shock is also associated with a reduction in expected volatility, the stochastic discount factor of the representative agents drops more than under the model without time-varying volatility. The steeper drop of the stochastic discount factor counterbalances the milder increase in investment and excess returns due to the volatility channel.

## 5. Conclusion

This study has examined the intertemporal distribution of productivity risk. Focusing on post-war US data, I have shown that the conditional mean of productivity growth is time-varying and extremely persistent. These low-frequency fluctuations strongly affect asset prices and macroeconomic quantities. My proposed production-based asset pricing model

[8] In Fig. 5(c), I plot conditional variances, $\widehat{\sigma_t^2}$. The StD.$[\sqrt{\widehat{\sigma}_t^2}/]E[\sqrt{\widehat{\sigma}_t^2}]$ ratio in the data is in the order of 10%.

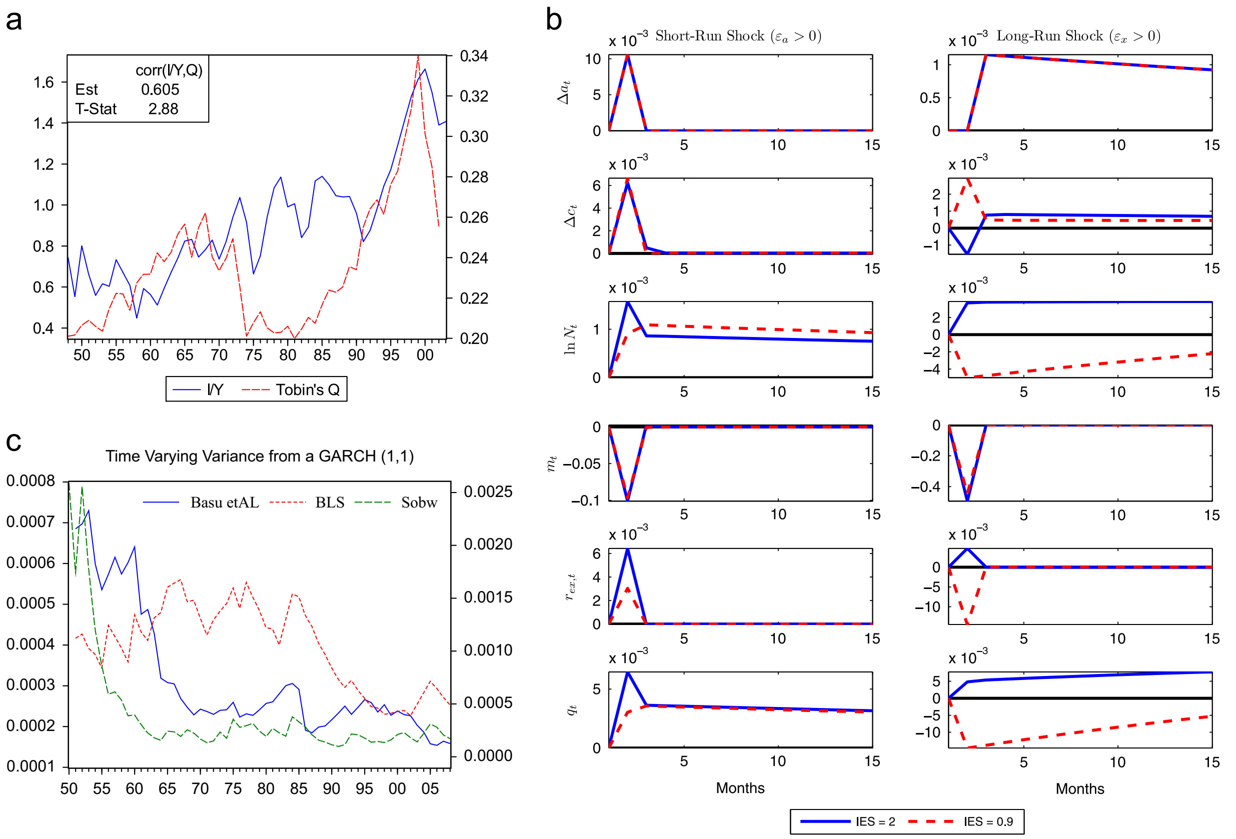


**Fig. 5.** Facts and frictions. (a) Tobin's Q and I/Y in the US, (b) reallocation costs and IES, and (c) time-Varying variance in US productivity. Panel A shows the Tobin's Q (dashed line, left-hand scale) and the investment-output ratio (solid line, right scale) in the US. The *t*-statistic of the estimated correlation is computed using Newey–West adjusted standard errors. Panel B shows monthly log-deviations from the steady state for the model with convex reallocation costs. Excess returns are not levered. All parameters are calibrated to the values reported in Table 3, except those of the adjustment cost function. The reallocation cost parameters are set as follows: $\epsilon = 100$; $a_1 = 9.e^{-6}$; and $\tau = 1.5$. Panel C shows GARCH(1,1)-based estimates of conditional variance for the three measures of US productivity growth used in Table 1. The right scale refers to the Solow residual measure.

features long-run uncertainty, convex adjustment costs, and recursive preferences à la Epstein-Zin. The model accounts for several key features of both prices and macroeconomic quantities, including consumption, investment, and output. A detailed examination of the role of the intertemporal elasticity of substitution, long-run productivity risk, and adjustment costs in this type of economy is provided as well.

There are a number of directions in which this work could be fruitfully extended. First, this paper assumes a frictionless labor market in order to keep the analysis as simple as possible. However, long-run labor fluctuations can have an important interaction with frictions (see, for example, Favilukis and Lin, 2011; Kuehn et al., 2010) and should be taken into account in future research. Second, there are good reasons to believe that these long-run components are endogenous (Kung and Schmid, 2010) and should be embodied in the analysis of both fiscal and monetary policies. Finally, long-run shocks should be added to cross-sectional studies (see, among others, Belo et al., 2012; Gomes et al., 2006).

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## Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2014.04.001.

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