# Baseline RBC model with labor market frictions

## 1 Households

The representative households solves an intertemporal allocation problem. To incorporate the motive of consumption smoothing, capital is assumed. Further, utility is supposed to depend only on consumption (and not, for example, on hours worked):

$$E_t \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{1}$$

The expenditure side of the budget constraint includes consumption $c_t$ and investments $i_t$. The income side consists of wage earnings $w_t n_t$ and interest payments from capital $r_t k_{t-1}$. Thus, the budget constraint reads

$$c_t + i_t = n_t w_t + r_t k_{t-1}, \tag{2}$$

and law of motion for capital

$$k_t = (1-\delta) k_{t-1} + i_t. \tag{3}$$

The maximization problem can be written as

$$\max_{c_t, k_t} E_t \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} \quad \text{w.r.t.} \quad c_t + k_t - (1-\delta) k_{t-1} = n_t w_t + r_t k_{t-1} \tag{4}$$

Deriving and combining the partial derivatives yields the so-called Euler equation for consumption:

$$c_t^{-\sigma} = \beta E_t c_{t+1}^{-\sigma} (r_{t+1} + 1 - \delta) \tag{5}$$

## 2 Labor market

The number of successful job matches $m_t$ is determined by a Cobb-Douglas matching function:

$$m_t = \gamma u_t^{\xi} v_t^{1-\xi}, \tag{6}$$

where $u_t$ denotes the number of unemployed workers and $v_t$ the numbers of open vacancies, respectively. The parameter $\gamma$ describes the matching efficiency and $\xi$ the elasticity of the matching function with respect to unemployment. The probability of filling a vacancy $q_t$ is the number of matches $m_t$ over the number of vacancies $v_t$

$$q_t = \frac{m_t}{v_t}. \tag{7}$$

Labor market tightness $\theta_t$ is defined as vacancies $v_t$ over unemployed $u_t$

$$\theta_t = \frac{v_t}{u_t}. \tag{8}$$

Each period an exogenous fraction $\rho_x$ of workers is separated from their jobs. Thus, the number of employed workers $n_t$ evolves according to:

$$n_t = (1 - \rho_x)(n_{t-1}) + m_{t-1} \tag{9}$$

The law of motion for the number of unemployed is consequently given by

$$u_t = 1 - n_{t-1}. \tag{10}$$

# 3 Firms

Firms produce output according to the Cobb-Douglas production function[1]

$$y_t = a_t k_t^{\alpha}. \tag{11}$$

The maximization problem of the firms implies that the optimal capital is chosen to equate the marginal product of capital to its price

$$r_t = \alpha a_t k_t^{\alpha-1}. \tag{12}$$

The value of a filled vacancy $J_t$ depends on the difference between the firm's output $y_t$ and the wage $w_t$ it has to pay. It also depends on the value of a filled vacancy tomorrow $J_{t+1}$ and the value of an unfilled vacancy tomorrow $V_{t+1}$

$$J_t = y_t - w_t - r_t k_t + \beta E_t \left[ \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} (1 - \rho_x) J_{t+1} + \rho_x V_{t+1} \right]. \tag{13}$$

The value of an unfilled vacancy can be further specified as

$$V_t = -\kappa + \beta E_t \left[ \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} (1 - \rho_x) q_t J_{t+1} + (1 - (1 - \rho_x) q_t) V_{t+1} \right]. \tag{14}$$

The parameter $\kappa$ describe vacancy posting costs. Using that in equilibrium it must hold that $V_t = 0$, one can derive the so-called Job Creation Condition:

$$\frac{\kappa}{q_t} = (1 - \rho_x) \beta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \left[ y_{t+1} - w_{t+1} - r_{t+1} k_t + \frac{\kappa}{q_{t+1}} \right] \tag{15}$$

[1]Note that the production function does not depend on hours worked since they also do not matter for the utility of the household

# 4 Deriving the wage function

The value of an employed worker $W_t$ is determined by the wage and value of being unemployed tomorrow $U_{t+1}$ and being employed tomorrow $W_{t+1}$

$$W_t = w_t + \beta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \left[ (1-\rho_x) W_{t+1} + \rho_x U_{t+1} \right]. \tag{16}$$

The value of an unemployed worker $U_t$ depends on the safe outside option $b$, and again, on tomorrows asset values:

$$U_t = b + \beta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \left[ \theta_t q_t (1-\rho_x) W_{t+1} + [1 - \theta_t q_t (1-\rho_x)] U_{t+1} \right]. \tag{17}$$

The wage of workers are chosen to maximize the Nash product

$$\max_{w_t} \quad (1-\eta)(W_t - U_t) = \eta(J_t - V_t), \tag{18}$$

where $\eta$ describes the worker's bargaining power. The FOC of this maximization yields the wage equation:

$$w_t = \eta(y_t - r_t k_t + \kappa\theta_t) + (1-\eta) b \tag{19}$$

# 5 Closing the model

The accounting identity reads:

$$y_t = c_t + k_t + \kappa v_t \tag{20}$$

Additionally, TFP follows a simple AR(1) process:

$$a_t = \rho a_{t-1} + \epsilon_t \tag{21}$$