D_t is debt, with trend X_t^D, $\frac{X_t^D}{X_{t-1}^D} = g_t^D$, $g_t^D = \rho_D \cdot g_{t-1}^D + \varepsilon_t^D$

Y_t is output, with trend X_t^Y, $\frac{X_t^Y}{X_{t-1}^Y} = g_t^Y$, $g_t^Y = \rho_Y \cdot g_{t-1}^Y + \varepsilon_t^Y$

ϕ_t is stationary marginal cost.

Assume all costs are from debt, since D_t and Y_t have different trend:

$$\frac{X_t^D}{X_t^Y} \cdot (\phi_t Y_t) = D_t$$

So that after detrending

$$\phi_t \cdot y_t = d_t.$$

This equation is independent from other equations, since d_t does NOT appear in any other place in the model (except for measurement equation if use debt data).

And AR(1) process: $g_t^D = \rho_D \cdot g_{t-1}^D + \varepsilon_t^D$ does not appear in model at all since debt is only intra-temporal.

Measurement equation:

$$\text{Debt\_obs} = \Delta \ln(D_t) = \ln D_t - \ln D_{t-1} = \ln d_t \cdot X_t^D - \ln d_{t-1} X_{t-1}^D$$

$$= \ln d_t - \ln d_{t-1} + \ln g_t^D$$

$$= \ln d_t - \ln \bar{d} - (\ln d_{t-1} - \ln \bar{d}) + \ln g_t^D - \ln \overline{g^D} + \ln \overline{g^D}$$

$$= \hat{d}_t - \hat{d}_{t-1} + \hat{g}_t^D + \ln \overline{g^D}$$ ("^" percentage deviation from SS)

So need to estimate ρ_D and std of ε_t^D, which are only appearing in measurement equation. Is this a problem?