1 The Model

1.1 Households

The economy is populated by infinitely many identical households of measure unity. The representative household is endowed with one unit of time and transfer from the government, $\tau_t F_t$, each period. Here τ_t is a share of natural resource revenues transferred to households by the fiscal authority. F_t represents total oil endowment meaning that fluctuations in world prices of oil or changes in natural resource exports has a direct impact on it. The time endowment is split between leisure and work. The representative household enters each period with a nominal money balance from the previous period (M_{t-1}), and receives profit from the production sector (Π_t), interest on fixed capital (K), and wages on supplied labor (L).

The representative household who has preferences over consumption goods (C_t), leisure ($1 - L_t$), and real money balances ($\frac{M_t}{P_t}$), seeks to solve the following maximization problem

$$\underset{\{C_t^M, C_t^N, M_t, L_t, K_{t+1}\}}{Max} E_0 \sum_{t=0}^{\infty} \beta^t \left[\zeta Ln(C_t) + \chi Ln(\frac{M_t}{P_t}) + \psi Ln(1 - L_t) \right], \tag{1}$$

subject to budget constraint

$M_{t-1} + \frac{1}{e_t}\tau_t F_t + W_t L_t + R_t K_t + P_t^M(1-\delta)K_t + \Pi_t = P_t^M K_{t+1} + M_t + P_t^M C_t^M + P_t^N C_t^N.$

Here β is the discount factor, C_t is the aggregate consumption index consisting of the consumption of manufactured goods C_t^M and non-tradable goods C_t^N, defined by $C_t = \frac{(C_t^M)^\theta (C_t^N)^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}}$, e_t is the nominal exchange rate in the units of foreign currency per unit of domestic currency, W_t is the nominal wage, R_t is the nominal interest rate, K_t is the capital, δ is the depreciation rate, P_t^N is the price of domestic non-tradable goods and $\zeta, \chi, \psi > 0$, $0 < \delta < 1$.

The price of foreign tradable goods in foreign currency ($P_t^{M^*}$) is given exogenously, hence we normalize it to one. Therefore, the price of tradable goods in the domestic currency (P_t^M) equals the inverse of nominal exchange rate: $P_t^M = \frac{1}{e_t} P_t^{M^*} = \frac{1}{e_t}$. Given the structure of the consumption aggregate, the consumption based price index is given by $P_t = (P_t^M)^\theta (P_t^N)^{1-\theta}$ [1].

1.2 Firms

There are two production sectors in the domestic economy, tradable an nontradable, and a continuum of identical firms in each sector. We assume Cobb-Douglas technology in both sectors with different capital/labor intensities and total factor productivities. The factors of production are mobile between the sectors, hence wages and the interest rates are equal in both sectors. We assume that capital can be produced in the tradable production sector

[1] The aggregate price index is defined as a minimum expenditure price required to purchase one unit of composite real consumption. For the derivation see Appendix 5.

or purchased from abroad in exchange to oil revenues. Capital is mobile between sectors, therefore, $K_t = K_t^M + K_t^N$.

A representative firm solves the profit maximization problem in the tradable sector:

$$\underset{\{K_t^M, L_t^M\}}{Max} \left\{ P_t^M Y_t^M - R_t K_t^M - W_t L_t^M \right\}, \tag{2}$$

and in the non-tradable sector:

$$\underset{\{K_t^N, L_t^N\}}{Max} \left\{ P_t^N Y_t^N - R_t K_t^N - W_t L_t^N \right\}. \tag{3}$$

Production functions are given by $Y_t^M = A(K_t^M)^\alpha (L_t^M)^{1-\alpha}$ and $Y_t^N = B(K_t^N)^\gamma (L_t^N)^{1-\gamma}$, where α and γ are capital shares, and A and B are total factor productivities in the tradable and non-tradable sectors, respectively.

1.3 Fiscal Authority

Each period the fiscal authority receives oil revenues and decides what share to transfer to households (denoted with $\tau_t F$) and what share to save in a special welfare fund (denoted as $(1-\tau_t)F$). For the sake of simplicity we assume that the fiscal authority faces two possible choices: (i) it is either disciplined ($\tau_t \neq 1$); or (ii) it behaves in an undisciplined way ($\tau_t = 1$).

As we see, a disciplined fiscal policy does not necessarily imply zero transfers to households and accumulation of all revenue from oil exports. Specifically, under a disciplined policy, instead of $\tau_t = 0$, (i.e., the government saves all the oil revenue), we assume $\tau_t \neq 1$, which implies that the fiscal authority constantly changes its control variable in order to maintain transfers to households at a permanent level. In other words, the fiscal authority chooses values for τ_t such that households' endowment does not deviate from the long run average value of oil revenues. This implies that there is no effect of temporary changes in oil revenues on the domestic economy and permanent income level is maintained. In contrast, under an undisciplined fiscal policy the fiscal authority transfers all oil revenues to the households and any shock to the oil revenue is reflected in the household's oil endowment.

We assume that the fiscal authority saves oil revenues outside the domestic economy without interest accumulation in a special welfare fund. The accumulation of the resources in this fund is determined by

$\Phi_t = \Phi_{t-1} + (1-\tau_t)F_t.$

1.4 Monetary Authority

The central bank chooses between one of three monetary regimes: (i) exchange rate targeting, (ii) price level targeting, and (iii) laissez faire, where the central bank fixes the money supply. The assumption of the three pure regimes may seem unrealistic, however, pegging the exchange rate and inflation targeting are two alternative policies usually considered by central bankers. Here we also consider a laissez faire policy or fixed money supply to capture the benchmark case where the central bank is inactive. Depending on the implemented policy rule, one of the variables, e_t, P_t, or M_t is fixed.

The central bank uses foreign exchange interventions to control the money supply. It can sell/buy domestic currency (M_t), and buy/sell some share (ω_t) of foreign exchange inflows ($\tau_t F_t$) in the foreign exchange market. This policy determines the path of international reserves (S_t) denominated in the foreign currency and held outside the domestic economy:

$S_t = S_{t-1} + \omega_t \tau_t F_t.$

An increase in the money supply is given by the following equation:

$\Delta M_t = \frac{1}{e_t} \omega_t \tau_t F_t.$

The foreign exchange interventions enable the monetary authority to play a crucial role in the allocation of resources in the economy. By increasing/decreasing the money supply, the central bank uses inflation tax and controls how much oil revenues are spent and how much are saved as international reserves.

Here we do not make any assumption about the interest income on accumulated reserves, neither of the fiscal authority, nor of the central bank. In the long run the equilibrium value of foreign reserves and consequently interest accumulation is zero. Introducing interest income does not have any qualitative impact in the long run, though one can use interest for estimating the optimal spending/saving strategy in the short run to get more precise quantitative results. Given heterogeneous reserve accumulation under different regimes, extra investment income would strengthen the position of a regime with higher accumulation from the welfare perspective.

1.5 Equilibrium

Now the characterization of the environment is completed, so we can define the equilibrium. Given the sequence of oil revenues $\{F_t\}_{t=0}^{\infty}$ there is an equilibrium where the sequence of household's choice of $\{C_t^M\}_{t=0}^{\infty}$, $\{C_t^N\}_{t=0}^{\infty}$, $\{M_t\}_{t=0}^{\infty}$, $\{L_t\}_{t=0}^{\infty}$,$\{K_{t+1}\}_{t=0}^{\infty}$ the firm's choice of $\{K_t^M\}_{t=0}^{\infty}$, $\{K_t^N\}_{t=0}^{\infty}$, $\{L_t^M\}_{t=0}^{\infty}$, $\{L_t^N\}_{t=0}^{\infty}$, the fiscal authority's choice of $\{\tau_t\}_{t=0}^{\infty}$, the central bank's control variable $\{\mu_t\}_{t=0}^{\infty}$, prices $\{P_t^M\}_{t=0}^{\infty}$, $\{P_t^N\}_{t=0}^{\infty}$, exchange rate $\{e_t\}_{t=0}^{\infty}$, interest rate $\{R_t\}_{t=0}^{\infty}$, and wage rate $\{W_t\}_{t=0}^{\infty}$ such that

(i) the households' utility maximization problem (1) is solved, (ii) the firms' profit maximization problems (2) and (3) are solved, (iii) the market clearing holds

- in the labor market: $(L_t)^s = (L_t^M)^d + (L_t^N)^d$;
- in the capital market: $(K_t)^s = (K_t^M)^d + (K_t^N)^d$;
- in the tradable goods market: $C_t^M + K_{t+1} - (1-\delta)K_t = A(K_t^M)^{\alpha}(L_t^M)^{1-\alpha} + \tau_t(1-\omega_t)F_t$;
- in the non-tradable goods market: $C_t^N = B(K_t^N)^{\gamma}(L_t^N)^{1-\gamma}$;

- in the money market: $M_t - M_{t-1} = \frac{1}{e_t}\omega_t \tau_t F_t$.

The market clearing condition in the tradable goods market means that households' demand for tradable goods can be either met by the domestic production of tradables or by import in exchange for foreign revenues from resource exports. Here ω_t may result in negative values meaning that spending on imported goods exceeds foreign revenues. This happens through a decrease in the money supply and correspondingly the depletion of international reserves.

1.6 Oil Revenues

Following the literature we assume that revenue from oil exports is determined by an AR(1) process defined as:

$$F_{t+1} = \rho F_t + (1-\rho)\overline{F} + \epsilon_t.$$

Here $0 < \rho < 1$ and $\epsilon \sim N(0, \sigma)$. The long run average of oil revenues ($\overline{F}$) is taken such that the economy produces mostly non-tradable goods and imports the main part of tradable goods from abroad. This assumption reflects the macroeconomic situation in developing economies heavily affected by the abundance of oil reserves.

Appendix 1. Solution of the Households' Problem

We can write the Lagrangian as:

$$L = \sum_{t=0}^{\infty} \beta^t \Bigg\{ \Bigg[\zeta Ln(\frac{(C_t^M)^\theta (C_t^N)^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}}) + \chi Ln(\frac{M_t}{P_t}) + \psi Ln(1-L_t) \Bigg]$$
$$+ \lambda_t \Bigg[M_{t-1} + \frac{1}{e_t}\tau_t F_t + W_t L_t + R_t K_t + P_t^M (1-\delta) K_t - P_t^M K_{t+1} - M_t - P_t^M C_t^M - P_t^N C_t^N \Bigg] \Bigg\}. \tag{4}$$

Now we can derive the first order conditions:

$$[C_t^M] : \beta^t \zeta \theta \frac{1}{C_t^M} - \beta^t \lambda_t \frac{1}{e_t} = 0 \tag{5}$$

$$[C_t^N] : \beta^t \zeta (1-\theta) \frac{1}{C_t^N} - \beta^t \lambda_t P_t^N = 0 \tag{6}$$

$$[M_t] : \beta^t \chi \frac{1}{M_t} + \beta^{t+1} \lambda_{t+1} - \beta^t \lambda_t = 0 \tag{7}$$

$$[L_t] : -\beta^t \psi \frac{1}{1-L_t} + \beta^t \lambda_t W_t = 0 \tag{8}$$

$$[K_{t+1}] : \beta^{t+1} \lambda_{t+1} R_{t+1} + \beta^{t+1} \lambda_{t+1} P_{t+1}^M (1-\delta) - \beta^t \lambda_t P_t^M = 0 \tag{9}$$

$$[\lambda_t] : M_{t-1} + \frac{1}{e_t}\tau_t F_t + W_t L_t + R_t K_t + P_t^M (1-\delta) K_t - P_t^M K_{t+1} - M_t - P_t^M C_t^M - P_t^N C_t^N = 0 \tag{10}$$

Further simplification gives us the following equations:

$$\frac{\zeta \theta e_t}{C_t^M} = \lambda_t \tag{11}$$

$$\frac{1-\theta}{P_t^N C_t^N} = \frac{\theta e_t}{C_t^M} \tag{12}$$

$$\frac{\chi}{M_t} = \frac{\zeta \theta e_t}{C_t^M} - \beta \frac{\zeta \theta e_{t+1}}{C_{t+1}^M} \tag{13}$$

$$\frac{\psi}{1 - L_t} = \frac{\zeta \theta e_t}{C_t^M} W_t \tag{14}$$

$$\frac{\beta e_{t+1} R_{t+1}}{C_{t+1}^M} + \frac{\beta(1-\delta)}{C_{t+1}^M} - \frac{1}{C_t^M} = 0 \tag{15}$$

$$M_{t-1} + \frac{1}{e_t} \tau_t F_t + W_t L_t + R_t K_t + P_t^M (1-\delta) K_t - P_t^M K_{t+1} - M_t - P_t^M C_t^M - P_t^N C_t^N = 0 \tag{16}$$

Appendix 2. Solution of the Firms' Problem

Maximization of the firms' problem yields

$$\underset{\{K_t^M, L_t^M\}}{Max} \left\{ P_t^M A (K_t^M)^{\alpha} (L_t^M)^{1-\alpha} - R_t K_t^M - W_t L_t^M \right\}; \tag{17}$$

$$[K_t^M] \; : \; \alpha P_t^M A (K_t^M)^{\alpha-1} (L_t^M)^{1-\alpha} = R_t; \tag{18}$$

$$[L_t^M] \; : \; (1-\alpha) P_t^M A (K_t^M)^{\alpha} (L_t^M)^{-\alpha} = W_t; \tag{19}$$

$$\underset{\{K_t^N L_t^N\}}{Max} \{ P_t^N B (K_t^N)^{\gamma} (L_t^N)^{1-\gamma} - R_t K_t^N - W_t L_t^N \}; \tag{20}$$

$$[K_t^N] \; : \; \gamma P_t^N B (K_t^N)^{\gamma-1} (L_t^N)^{1-\gamma} = R_t; \tag{21}$$

$$[L_t^N] \; : \; (1-\gamma) P_t^N B (K_t^N)^{\gamma} (L_t^N)^{-\gamma} = W_t. \tag{22}$$

Equating (17) with (20) and (18) with (21) we get

$$\alpha P_t^M A (K_t^M)^{\alpha-1} (L_t^M)^{1-\alpha} = \gamma P_t^N B (K_t^N)^{\gamma-1} (L_t^N)^{1-\gamma}, \tag{23}$$

$$(1-\alpha) P_t^M A (K_t^M)^{\alpha} (L_t^M)^{-\alpha} = (1-\gamma) P_t^N B (K_t^N)^{\gamma} (L_t^N)^{-\gamma} \tag{24}$$

We can write the last two equations as:

$$\frac{K_t^M / L_t^M}{K_t^N / L_t^N} = \frac{(1-\gamma) / \gamma}{(1-\alpha) / \alpha}. \tag{25}$$

Appendix 3. Structural form of the model

- endogenous variables: C_t, C_t^M, C_t^N, P_t^M, P_t^N, K_t, K_t^M, K_t^N, L_t, L_t^M, L_t^N, R_t, W_t, Φ_t, S_t, ω_t, and two of the three variables e_t, P_t or M_t depending on the monetary policy target.
- exogenous variables: F_t, τ_t, and one of the three variables e_t, P_t or M_t depending on the monetary policy target.
- parameters: α, γ, β, ζ, θ, χ, ψ, δ, ρ, A, B.

1. $C_t = \frac{(C_t^M)^\theta (C_t^N)^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}}$
2. $P_t^M = \frac{1}{e_t}$
3. $P_t = (P_t^M)^\theta (P_t^N)^{1-\theta}$
4. $\frac{P_t^M C_t^M}{P_t^N C_t^N} = \frac{\theta}{1-\theta}$
5. $\frac{\chi}{M_t} = \frac{\zeta \theta e_t}{C_t^M} - \beta \frac{\zeta \theta e_{t+1}}{C_{t+1}^M}$
6. $\frac{\psi}{1-L_t} = \frac{\zeta \theta e_t}{C_t^M} W_t$
7. $R_{t+1} = \frac{C_{t+1}^M}{\beta e_{t+1} C_t^M} - \frac{1-\delta}{e_{t+1}}$
8. $C_t^M + K_{t+1} - (1-\delta) K_t = A (K_t^M)^\alpha (L_t^M)^{1-\alpha} + \tau_t (1-\omega_t) F_t$
9. $C_t^N = B (K_t^N)^\gamma (L_t^N)^{1-\gamma}$
10. $\alpha P_t^M A (K_t^M)^{\alpha-1} (L_t^M)^{1-\alpha} = R_t$
11. $\gamma P_t^N B (K_t^N)^{\gamma-1} (L_t^N)^{1-\gamma} = R_t$
12. $(1-\alpha) P_t^M A (K_t^M)^\alpha (L_t^M)^{-\alpha} = W_t$
13. $(1-\gamma) P_t^N B (K_t^N)^\gamma (L_t^N)^{-\gamma} = W_t$
14. $\Phi_t = \Phi_{t-1} + (1-\tau_t) F_t$
15. $M_t = M_{t-1} + \frac{1}{e_t} \omega_t \tau_t F_t$
16. $S_t = S_{t-1} + \omega_t \tau_t F_t$
17. $K_t = K_t^M + K_t^N$
18. $L_t = L_t^M + L_t^N$

Appendix 4. Steady State

- endogenous variables: C, C^M, C^N, P^M, P^N, L, L^M, L^N, K, K^M, K^N, W, R, and two of the three variables e, P or M depending on the monetary policy target.
- exogenous variables: F and one of the three variables e, P or M depending on the monetary policy target.
- parameters: α, γ, β, ζ, θ, χ, ψ, δ, ρ, A, B.

1. $C = \frac{(C^M)^\theta (C^N)^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}}$
2. $P^M = \frac{1}{e}$
3. $P = (P^M)^\theta (P^N)^{1-\theta}$
4. $\frac{P^M C^M}{P^N C^N} = \frac{\theta}{1-\theta}$
5. $\frac{\chi}{M} = \frac{\zeta \theta e}{C^M}(1-\beta)$
6. $\frac{\psi}{1-L} = \frac{\zeta \theta e}{C^M} W$
7. $R = \frac{1}{\beta e} - \frac{1-\delta}{e}$
8. $C^M + \delta K = A(K^M)^\alpha (L^M)^{1-\alpha} + F$
9. $C^N = B(K^N)^\gamma (L^N)^{1-\gamma}$
10. $\alpha P^M A(K^M)^{\alpha-1}(L^M)^{1-\alpha} = R$
11. $\gamma P^N B(K^N)^{\gamma-1}(L^N)^{1-\gamma} = R$
12. $(1-\alpha)P^M A(K^M)^\alpha (L^M)^{-\alpha} = W$
13. $(1-\gamma)P^N B(K^N)^\gamma (L^N)^{-\gamma} = W$
14. $K = K^M + K^N$
15. $L = L^M + L^N$

Appendix 5. Derivation of the Price Index

The consumption-based price index solves following minimization problem:

$P_t C_t = Min(P_t^M C_t^M + P_t^N C_t^N);$

$s.t. \frac{(C_t^M)^\theta (C_t^N)^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}} = 1.$

The first order conditions yield

$\frac{(1-\theta)}{\theta} \frac{P_t^M}{P_t^N} = \frac{C_t^N}{C_t^M}.$

In the optimal solution we can write

$P_t \frac{(C_t^M)^\theta (C_t^N)^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}} = P_t^M C_t^M + P_t^N C_t^N.$

Dividing both sides by C_t^M gives us

$P_t \frac{(C_t^M)^{\theta-1} (C_t^N)^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}} = P_t^M + P_t^N \frac{C_t^N}{C_t^M}.$

using FOC we get

$P_t \frac{(\frac{(1-\theta)}{\theta} \frac{P_t^M}{P_t^N})^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}} = P_t^M + P_t^N \frac{(1-\theta)}{\theta} \frac{P_t^M}{P_t^N}.$

After some simplification we end up with the equation for aggregate price index:

$P_t = (P_t^M)^\theta (P_t^N)^{1-\theta}.$