Ageing and Pensions in General Equilibrium: Labor Market Imperfections Matter*

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Abstract

This paper reexamines the effects of population ageing and pension reforms in an OLG model with labour market frictions and endogenous early retirement decisions. Except for labour market imperfections, our model shares many features of existing models of population ageing. In our setup, even with a perfectly inelastic labour supply, population ageing will in general affect equilibrium (un-)employment via its impact on factor prices. In order to focus on labour market imperfections and their consequences, we do not at this stage build a multi-country model. We consider a single country, and illustrate the potential impact of international capital flows by comparing two polar cases, the closed vs small open economy case. Shocks leading to a drop in the interest rate, such as an increase in life expectancy, or a reform of the social security system towards better funded pension schemes, also lead to a drop in the unemployment rate. Lower capital costs stimulate labour demand and induces firm to post more vacancies. The consequence is a larger employment rate and larger bargained wages. The employment effects are reinforced by the increased participation of senior workers, induced both by the larger probability to find a job and the larger wage. Neglecting the interactions between pension reforms and labour market frictions leads to an underestimation of the effects of such reforms.

Keywords: Overlapping Generations, Search Unemployment, Labor Force Participation, Ageing, Pensions, Labor Market

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1 Introduction

Population ageing results from three factors: declining fertility, increasing longevity, and the retirement of the baby-boom generations, especially in Europe. Such changes raise concern about the sustainability of existing public pension schemes and suggest that pension reforms will at some point become unescapable. There is already a vast literature aiming at quantitative evaluations of the impact of population ageing and of various pension reforms. OLG models à la Auerbach-Kotlikoff (1987) (deterministic setting) and Rios-Rull (1996) (stochastic set-up) provide a most useful setup wherein to analyze such developments and questions. All studies point out that, with relatively scarcer labour, equilibrium factor prices will change, wages will rise and interest rates fall. Evaluating correctly the magnitude of such effects and all their implications is crucial for policy analysis and recommendations.

Recent contributions using this dynamic general equilibrium approach have emphasized the role of international capital markets (see for instance Börsch-Supan et al.,(2006), Attanasio et al.(2007) and the references therein). Life-cycle models imply that changes in the age-structure of a population has an impact on aggregate savings. Because the ageing process is not synchronized at the world level (ageing is already well underway in the "North", while only starting in the "South"), saving rates will not be affected in all countries in the same way nor at the same time. In a world with capital mobility, one should thus expect substantial international capital flows and take them into account when trying to quantify expected interest rate and wage changes. This concern has motivated the development of multi-country models. The modelling of the labor market has however remained quite streamlined. It is typically based on the perfect competition setup. Models with endogenous labor supply focus on individual hours of work rather than employment. As noted by Lucas (2007), neglecting labour adjustments along the extensive margin may seriously bias the results of some policy scenarios. Adjustments along the extensive margin can play a significant role, especially in countries (like many European ones) where labor market imperfections and institutions can have a strong impact on search and participation behaviors.

This paper aims at introducing a more realistic representation of the labour market by combining life cycle features and labour market frictions à la Mortensen-Pissarides in a single framework. In this setup, even with a perfectly inelastic labour supply, population ageing will in general affect equilibrium (un-)employment via its impact on factor prices. Our objective is to assess the quantitative importance of this channel, both for the sustainability of current pension schemes and the effectiveness of pension reforms. We shall furthermore allow for early retirement provisions and their impact on the participation rate of elderly workers. This is motivated by empirical studies showing that financial incentives do have a significant impact on early retirement decisions (see for instance Gruber and Wise (1999) or Duval (2003) and the references therein). Although the official retirement age is exogenous in most countries, the actual average retirement age is endogenous and contributes to explain the lower employment rate of older workers.

Except for labour market imperfections, our model shares many features of existing models of population ageing. It is worth pointing out some specificities. Borsch-Supan et al. (2006) introduce endogenous labour supply and calibrate leisure utility parameters essentially to capture the effects of lower senior participation rates. Attanasio et al. (2007) suggest that, although accounting for international capital flows is crucial to quantify the effects of pension reforms on price and output changes, it may have little impact of the fiscal variables themselves. This may happen because of counteracting effects (for instance, lower capital tax revenues are compensated by higher wage tax revenues following a increase in payroll or consumption taxes), but need not be generally true. Kruger-Ludwig (2007) introduce idiosyncratic shocks and precautionary savings, and focus on redistributional and welfare effects. Heijdra and Romp (2007) give special emphasis to endogenous early retirement decisions. The optimal retirement age is affected by longevity. It is shown that increased longevity is most likely to increase the optimal retirement age, although the effect is a priori ambiguous. Börsch-Supan and Ludwig (2010) examine more carefully the role of the labour market and the connections/interactions between pension reforms and labour market reforms. They distinguish labour supply adjustments along both the extensive and the intensive margins. Still, their representation of the labour market remains highly-stylized. Models combining life cycle features and labour market frictions have already been used, although in a totally different perspective. Hairault and Langot (2008) focus on incomplete financial markets and income inequality. They show that lower replacement rates are detrimental to the consumption of the retirees. Hairault et al. (2010) examine early retirement decisions and job search intensities in a search model à la McCall (1970), extended to include stochastic ageing. They emphasize the effect of the distance from retirement age on the financial incentives to invest in search activities. Chéron et al. (2008a,b) further develop this modeling strategy and replace the assumption of an exogenous wage distribution by a matching process à la Mortensen-Pissarides (1994). In these models, the labour force is assumed to be constant and the retirement age is deterministic. There is no post-retirement consumption and no savings.

The model developed in this paper has the following main building blocks (i) demographics; (ii) labour market frictions and institutions (including early retirement provisions); (iii) life-cycle consumption and savings. In order to focus on labour market imperfections and their consequences, we do not at this stage build a multi-country model. We consider a single country, and illustrate the potential impact of international capital flows by comparing two polar cases, the closed vs small open economy case. Rios-Rull (2001) emphasized the importance of a careful description of the demographic process. The latter can vary a lot from country to country. To fit actual data, we use French mortality, fertility and migration rates, so that the size of each cohort in every period coincides with the most recent demographic projections. There is no aggregate uncertainty. The sole source of uncertainty is age-dependent mortality. In line with most of the literature, we assume that there is a perfect annuity market allowing individuals to fully insure against life uncertainty. As in de la Croix et al. (2007), we use generational accounting studies to link taxes and public expenditures to demographic changes. We assume however exogenous human capital and introduce instead labour market frictions with non age-directed search by firms. Early retirement is endogenous and decided by households; it is a function of the generosity of early retirement schemes.

The model is calibrated (mostly on French data) and simulated. We first examine how labour market imperfections change the mechanisms by which demographic shocks are expected to affect the economy over the 21st century. The main message brought by the simulations is that shocks leading to a drop in the interest rate, such as an increase in life expectancy, or a reform of the social security system towards better funded pension schemes, also lead to a drop in the unemployment rate. This is because lower capital costs stimulate labour demand and induces firm to post more vacancies. The consequence is a larger employment rate and larger bargained wages. The employment effects are reinforced by the increased participation of senior workers, induced both by the larger probability to find a job and the larger wage. We next reevaluate the interactions between pension reforms and labour market frictions. We focus on two scenarios, a reduction in the generosity of early retirement provisions and a shift to a fully-funded scheme. It is shown that neglecting labour market frictions and equilibrium employment rate changes leads to an underestimation of the effects of such reforms.

The model is described in Section 2. The calibration is detailed in section 3. Simulation results are presented and discussed in section 4. Section 5 concludes.

2 The Model

We develop a dynamic general equilibrium model with overlapping generations and frictions on the labour market. We use to represent the latter a setup à la Diamond-Mortensen-Pissarides (DMP), with (exogenous) job destruction and a matching function. To simplify, we assume perfect substitutability between all workers, although they differ by their age and experience. Perfect substitutability means that there is a single matching function (all vacancies can be filled by any worker, whatever his age). Bargained wages will reflect differences in work efficiency. Notice that age-directed search is not a credible strategy in our setup. Because the value of an unfilled vacancy is zero, a firm which would open a vacancy targeted say on young workers would eventually hire the first worker she meets, provided the surplus to be shared is positive.

2.1 Demographics

We consider an overlapping generation model where each member of a generation can live for up to sixteen five-year periods (from age 25 till 104), indexed from 0 to 15. Let $Z_{a,t}$ denote the size of the generation reaching age a at period t. The size of new generations changes over time at an exogenous rate x_t:

$$Z_{0,t} = (1 + x_t)\, Z_{0,t-1}, \quad \forall t > 0, \tag{1}$$

where x_t includes both fertility and migration shocks at age zero. Abstracting from later migration shocks, the size of a given generation t declines deterministically through time. This size is determined by a cumulative survival probability $\beta_{a,t+a}$ so that:

$$Z_{a,t+a} = \beta_{a,t+a}\, Z_{0,t} + X_{a,t+a}, \quad \forall a \in [15, 0), \tag{2}$$

where $0 \leq \beta_{a,t+a} \leq 1$ is decreasing in a, with $\beta_{0,t} = 1$. Migration flows after age 0 are taken into account through $X_{a,t+a}$. Total (adult) population at time t is equal to $Z_t = \sum_{a=0}^{15} Z_{a,t}$. The demographic growth and survival probability vector can vary over time. These changes are assumed to be exogenous.

All people above 64 ($8 \leq a \leq 15$) are inactive, so that 65 is the mandatory retirement age. Variable $z_{a,t+a}$ is the participation rate, so that

$$P_{a,t+a} = z_{a,t+a}\, Z_{a,t+a} \tag{3}$$

is equal to the active population, broadly defined to include workers on an early retirement scheme. At time t, the participating members of generation of age a are either employed, unemployed, or on an early retirement scheme:

$$\begin{aligned} P_{a,t} &= N_{a,t} + U_{a,t} + E_{a,t}\,, \\ &= \left[n_{a,t} + u_{a,t} + e_{a,t} \right] P_{a,t}\,, \qquad 0 \leq a \leq 7 \\ &\Leftrightarrow \quad n_{a,t} + u_{a,t} + e_{a,t} = 1 \end{aligned} \tag{4}$$

Lower-case letters denote the proportion of individuals in each status. The early retirement rate before 55 is zero ($e_{a,t} = 0$ for $a < 6$). Let $\lambda_{6,t}$ denote the fraction of people who choose to retire early and leave the labor market at age $a = 6$ (between 55 and 60), so that the number of early retired workers of that age group is $E_{6,t} = \lambda_{6,t}\, P_{6,t}$. Similarly, let $\lambda_{7,t}$ denote the fraction of active workers who decide to leave the labor market at age $a = 7$ (between 60 and 64). The total number of workers on early retirement at time t is then equal to:

$$\begin{aligned} E_{6,t} + E_{7,t} &= e_{6,t}\, P_{6,t} + e_{7,t}\, P_{7,t}\,, \\ \text{with:} \quad & e_{6,t} = \lambda_{6,t}\,, \\ & e_{7,t} = \lambda_{6,t-1} + \lambda_{7,t}\,(1 - \lambda_{6,t-1})\,. \end{aligned} \tag{5}$$

2.2 Labor Market Flows

We use a DMP representation of search frictions on the labor market. We assume a constant returns to scale matching function. Except for early retirement decisions, job separations are determined by an exogenous job destruction rate χ. The pool of job seekers at time t is equal to the number of new entrants $P_{0,t}$ plus the unemployed job seekers in older age groups. Let us denote $\Omega_{a,t}$ the number of job seekers of age a at the beginning of period t. Given a mandatory retirement age of 65, the total number of job seekers at the beginning of period t, denoted Ω_t, is then equal to:

$$\begin{aligned}\Omega_t &= \sum_{a=0}^{7} \Omega_{a,t} \ , \\ &= P_{0,t} + \sum_{a=1}^{5} \left[1-(1-\chi)\, n_{a-1,t-1}\right] P_{a,t} \\ &\quad + \left[1-(1-\chi)\, n_{5,t-1}\right](1-\lambda_{6,t})\, P_{6,t} \\ &\quad + \left[(1-\lambda_{6,t-1})-(1-\chi)\, n_{6,t-1}\right]\, (1-\lambda_{7,t})\, P_{7,t}\end{aligned} \tag{6}$$

where parameters $\lambda_{6,t}$ and $\lambda_{7,t}$ introduce the effects of early retirement decisions made by households (see section 2.3 below). Let us also denote V_t the number of vacancies posted by firms at the beginning of period t. Given a matching function:

$$M_t = M(V_t, \Omega_t)\,, \tag{7}$$

the probabilities of finding a job and of filling a vacancy will be given respectively by:

$$p_t = \frac{M_t}{\Omega_t} \qquad \text{and} \qquad q_t = \frac{M_t}{V_t}\,. \tag{8}$$

The number of employed workers in age group a is determined by the sum of non-destroyed jobs (when $a > 0$) and new hires:

$$\begin{aligned} n_{a,t} &= p_t\, \frac{\Omega_{a,t}}{P_{a,t}}\,, && \text{for } a = 0\,; \\ &= (1-\chi)\, n_{a-1,t-1} + p_t\, \frac{\Omega_{a,t}}{P_{a,t}}\,, && \text{for } 1 \le a \le 5\,; \\ &= (1-\lambda_{a,t})\,(1-\chi)\, n_{a-1,t-1} + p_t\, \frac{\Omega_{a,t}}{P_{a,t}}\,, && \text{for } 6 \le a \le 7\,. \end{aligned}$$

After substituting for $\Omega_{a,t}$, this equation becomes:

$$\begin{aligned} n_{a,t} &= p_t\,, && \text{for } a = 0\,; \\ &= (1-p_t)(1-\chi)\, n_{a-1,t-1} + p_t\,, && \text{for } 1 \le a \le 5\,; \\ &= (1-p_t)(1-\lambda_{a,t})\,(1-\chi)\, n_{a-1,t-1} + p_t(1-\lambda_{a,t})\,, && \text{for } a = 6\,; \\ &= (1-p_t)(1-\lambda_{a,t})\,(1-\chi)\, n_{a-1,t-1} + p_t(1-\lambda_{a,t})(1-\lambda_{a-1,t-1})\,, && \text{for } a = 7\,. \end{aligned} \tag{9}$$

The same equation can be written in terms of the probability of filling a vacancy q_t by using $p_t = q_t V_t / \Omega_t$. Total employment will then be equal to:

$$N_t = \sum_{a=0}^{7} N_{a,t}\,, \qquad \text{with } N_{a,t} = n_{a,t}\, P_{a,t}\,.$$

2.3 Households

We assume an economy with state-contingent markets, so that each individual can fully insure against idiosyncratic risk at the beginning of his lifetime. Given a sequence of contingent wages and prices, an individual born at time t will determine his optimal contingent consumption plan by maximizing his expected utility, subject to his intertemporal budget constraint. In this setting, the individual optimization problem is identical to the optimization program of a hypothetical large household including all members of a given generation. Provided the instantaneous utility function is separable in consumption and leisure, all members of a given generation will have the same consumption level, whatever their employment or participation status.

One of our main objective is endogenize early retirement decisions and their impact of the aggregate supply of labor. In the context of OLG models, several theoretical contributions have paved the way. Hu (1979) focuses on the effect of social security on capital accumulation and labor supply in a PAYG case. Retirement decision in the second period of life is endogenous (individual chooses the fraction of time devoted to leisure). The short-run effects of pensions depend on the elasticities of demand and supply of labor; long-run effects are influenced by the elasticities of savings and bequest. Michel and Pestieau (1999) have a similar endogenous labor participation choice. The fraction of time spent working in the second period declines when the size of the social security increases. For first-best optimum to be achieved, both PAYG social security tax and the retirement age must be controlled for. de la Croix, Mahieu, and Rillaers (2004) analyze the consequences of demographic change and the impact of social security reforms (lower pensions, higher contributions, postponing retirement). The effect on the optimal retirement age of a drop in fertility depends on the parameters of preferences and technology.

There will be no attempt however to explain the evolution of participation rates across time or over the life cycle for people less than 55[1]. We thus assume exogenous participation rates $z_{a,t+a}$ and focus on early retirement decisions $e_{a,t+a}$, especially the impact of changes in the generosity of early retirement schemes, given the state of the labor market. We write the objective function

[1]See de la Croix et al. (2007) for further motivation of this choice.

of the household (effectively of one cohort) as follows:

$$W_t^H = \max_{c_{a,t+a},\, \lambda_{6,t+6},\, \lambda_{7,t+7}} \sum_{a=0}^{15} \beta^a \, \beta_{a,t+a} \left\{ \mathcal{U}(c_{a,t+a}) - d^n \, n_{a,t+a} \, z_{a,t+a} + d_a^e \, \frac{(e_{a,t+a})^{1-\phi}}{1-\phi} \, z_{a,t+a} \right\} Z_{0,t} \,, \tag{10}$$

where $Z_{0,t}$ is the initial size of the cohort, $\beta_{a,t+a}$ is a cumulative survival probability and β is a subjective discount factor[2]. This representation of preferences is similar to the one used by other authors considering endogenous participation decisions in a model with frictional unemployment (see for instance Den Haan - Kaltenbruner (2009) and references therein). The main differences come from the fact that our model includes life-cycle features and focuses on early retirement decisions. Instantaneous utility is assumed to be separable in c, n and e. The utility of per capita consumption is represented by a standard concave function (we shall use a logarithmic function). The marginal disutility of working is assumed to be constant[3], equal to d^n. The extra utility derived from early retirement is represented by a concave function of the early retirement rate ($0 < \phi < 1$). The decision variables are c, λ_6 and λ_7. These last two variables refer to the fraction of agents in the corresponding age groups who decide to go on early retirement and leave the labor market, respectively at age 55 and 60. Employment rates $n_{a,t+a}$ and early retirement rates $e_{a,t+a}$ (and their the connection to the λ's) are given by (5) and (9).

The household's flow budget constraint at time $t+a$ takes the form:

$$\begin{aligned} &\Big[(1-\tau^w_{t+a})\, w_{a,t+a} \,.\, n_{a,t+a} + b^u_{a,t+a} \,.\, u_{a,t+a} + b^e_{a,t+a} \,.\, e_{a,t+a} + b^i_{a,t+a} \,.\, i_{a,t+a}\Big] \,.\, z_{a,t+a} \\ &\qquad + \frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}} \, R_{t+a} \, s_{a-1,t+a-1} = (1+\tau^c_{t+a})\, c_{a,t+a} + s_{a,t+a} \end{aligned} \tag{11}$$

Wage and consumption tax rates are given by τ^w and τ^c respectively; $b^u_{a,t+a}, b^e_{a,t+a}, b^i_{a,t+a}$ are the replacement benefits received respectively by the unemployed, early retired or old-aged worker on a legal pension scheme ($i_{a,t+a}$ is a dummy variable equal to zero when $a < 65$, equal to 1 afterwards); $s_{a,t+a}$ is the financial wealth accumulated at time $t+a$, in per capita terms. This financial wealth is held either in the form of shares or physical capital. Because there is perfect insurance against individual life uncertainty (as if there were a perfect annuity market), the total return to savings is equal to the gross risk free interest rate R_{t+a} divided by the survival probability $\beta_{a,t}/\beta_{a-1,t-1}$.

The optimal consumption plan must satisfy the usual Euler equation:

$$\frac{u'_{c_{a,t+a}}}{1+\tau^c_{t+a}} = \beta \, R_{t+a+1} \, \frac{u'_{c_{a+1,t+a+1}}}{1+\tau^c_{t+a+1}} \,.$$

[2]As stressed by Ríos-Rull(2001), β can represent both pure time preference and the effect of family size changes (implying that consumption is enjoyed differently at different ages).

[3]Our formulation normalizes the disutility of search activities of the unemployed to zero. Setting $d^n \geq 0$ amounts to assuming that the disutility of working can be larger than that of searching.

After substitution and rearrangements, the condition determining the optimal proportion of early retired workers aged 60 can be shown to be:

$$\frac{b^e_{7,t+7}}{(1+\tau^c_{t+7})\, c_{7,t+7}} + d^e_7\, (e_{7,t+7})^{-\phi} = \pi_{7,t+7} \left[\frac{(1-\tau^w_{t+7})\, w_{7,t+7}}{(1+\tau^c_{t+7})\, c_{7,t+7}} - d^n\right] + (1-\pi_{7,t+7}) \left[\frac{b^u_{7,t+7}}{(1+\tau^c_{t+7})\, c_{7,t+7}}\right] , \tag{12}$$

where π is the unconditional probability of being employed (ie, the probability that an active worker chosen at random is actually employed). A similar condition holds for early retirement at age 55 (see details in appendix). Equation (12) says that the household's optimal early retirement rate is such that the marginal utility of early retirement (early retirement income plus leisure utility) is equal to the expected marginal utility of remaining active on the job market (wage income net of labor disutility and unemployment benefit, each weighted by their respective probability). Which member of the household will actually go on early retirement does not depend on the initial employment status. Both employed and unemployed workers may become early retirees. This follows from our specification of labor market flows (equation (9)). Imposing that only previously unemployed workers can shift to early retirement would be much too restrictive. Firms do take advantage of the generosity of early retirement schemes to adjust the number of their employees, while elderly workers do accept to quit jobs earlier if the early retirement compensation is appropriate.

For later use, we also note that the value of an additional job for a household of age a is given by:

$$\begin{aligned}\frac{1}{u'_{c_{a,t}}}\,\frac{\partial W^H_t}{\partial N_{a,t}} &= \frac{1}{u'_{c_{a,t}}}\,\frac{1}{z_{a,t}\, Z_{a,t}}\,\frac{\partial W^H_t}{\partial n_{a,t}} \\ &= \sum_{j=0}^{7-a} \frac{\beta_{a+j,t+j}}{\beta_{a,t}}\, \beta^j\, \frac{u'_{c_{a+j,t+j}}}{u'_{c_{a,t}}} \left\{ \frac{(1-\tau^w_{t+j})\, w_{a+j,t+j} - b^u_{a+j,t+j}}{(1+\tau^c_{t+j})} - \frac{d^n}{u'_{c_{a+j,t+j}}} \right\} \frac{\partial n_{a+j,t+j}}{\partial n_{a,t}}\end{aligned} \tag{13}$$

where $\partial n_{a+j,t+j}/\partial n_{a,t}$ can be obtained from (9).

2.4 Firms

There are two productive factors, labor and capital. Labor is measured in efficiency units. Efficiency may vary across age (because of experience) and across generations (because of education). We define the total labor input as follows:

$$H_t = \sum_{a=0}^{7} h_{a,t}.N_{a,t} \, .$$

We assume a constant return to scale production function in labor and capital:

$$Y_t = A_t\, F(K_t, H_t)\,, \tag{14}$$

where A_t stands for total factor productivity. Firms rent capital at cost $v_t = R_t + \delta - 1$ and pay a gross wage $w_{a,t}$ to workers of age a. We denote by ζ the employer wage tax. The representative firm maximizes the discounted value of all the dividends (profits) that will be distributed to her shareholders. Profits at time t are given by:

$$\Pi_t = F(K_t, H_t) - v_t\, K_t - \sum_{a=0}^{7} (1+\zeta_t)\, w_{a,t}\, N_{a,t} - \mathsf{a}\, V_t \tag{15}$$

where "a" stands for the cost of posting a vacancy. The value of the firm can thus be written as follows[4]:

$$W_t^F = \max_{K_t, V_t} \left\{ F(K_t, H_t) - v_t\, K_t - \sum_{a=0}^{7} (1+\zeta_t)\, w_{a,t}\, N_{a,t} - \mathsf{a}\, V_t \right\} + R_{t+1}^{-1}\, W_{t+1}^F \tag{16}$$

subject to (9) and $p_t = q_t\, V_t / \Omega_t$. The first-order optimality conditions are:

$$v_t = F_{K_t}\,, \tag{17}$$

$$\mathsf{a} = q_t \sum_{a=0}^{7} \frac{\Omega_{a,t}}{\Omega_t} \frac{\partial W_t^F}{\partial N_{a,t}}\,, \tag{18}$$

where $\frac{\partial W_t^F}{\partial N_{a,t}}$ is the value at time t of an additional worker of age a. With a job destruction rate χ, this value is equal to:

$$\begin{aligned}\frac{\partial W_t^F}{\partial N_{a,t}} &= \frac{1}{z_{a,t}\, Z_{a,t}} \frac{\partial W_t^F}{\partial n_{a,t}} \\ &= \sum_{j=0}^{7-a} \frac{\beta_{a+j,t+j}}{\beta_{a,t}}\; R_{t,t+j}^{-1} (1-\lambda_{a+j-1,t+j-1})\,(1-\lambda_{a+j,t+j})\,(1-\chi)^j \\ &\qquad . \left\{ h_{a+j,t+j}\, F_{H_{t+j}} - (1+\zeta_{t+j})\, w_{a+j,t+j} \right\}, \end{aligned} \tag{19}$$

where $\lambda_{a+i,t+i} \equiv 0$ for $a+i < 6$.

[4]Shareholders may belong to different age groups and have different consumption level. Still, they all have the same discount factor given by $\tilde{\beta}_{t+1} = \beta\, \frac{u'_{c a+1,t+1}}{u'_{c_{a,t}}} = R_{t+1}^{-1}\,,\ \forall a \in \{0, 14\}$.

2.5 Government

We assume that unemployment and (early or legal) retirement benefits are determined by an exogenous fraction of the relevant gross wage, so that

$$\begin{aligned} b^u_{a,t} &= \rho^u_t\, w_{a,t} && \text{for} \quad 0 \le a \le 7\,; \\ b^e_{a,t} &= \rho^e_t\, w_{a,t} && \text{for} \quad 6 \le a \le 7\,; \\ b^i_{a,t} &= \rho^i_t \sum_{i=0}^{3} \frac{w_{a-i,t-i}}{4} && \text{for} \quad 8 \le a \le 15\,. \end{aligned} \tag{20}$$

For retirement the benefit is computed on the average wage of the last four periods. Total transfer expenditures are then equal to:

$$T_t = \rho^u_t \sum_{a=0}^{7} w_{a,t}\, u_{a,t}\, z_{a,t}\, Z_{a,t} + \rho^e_t \sum_{a=6}^{7} w_{a,t}\, e_{a,t}\, z_{a,t}\, Z_{a,t} + \rho^i_t \sum_{i=0}^{3} \frac{w_{a-i,t-i}}{4} \sum_{a=8}^{15} z_{a,t}\, Z_{a,t}. \tag{21}$$

Public consumption is assumed to be a fraction of output, i.e.

$$G_t = \bar{g}\,(Y_t - \mathsf{a}\, V_t). \tag{22}$$

We further assume that the "government" balances its budget in every (five-year) period by adjusting consumption taxes (ie, τ^c_t is the adjusting variable)[5]:

$$\tau^c_t\, C_t + (\tau^w_t + \zeta_t) \left(\sum_a w_{a,t}\, n_{a,t}\, P_{a,t} \right) = G_t + T_t\,, \tag{23}$$

where aggregate consumption $C_t = \sum_a c_{a,t}\, Z_{a,t}$.

We abstract from public debt here. Public debt could be introduced by supposing an exogenous path of the debt, and keeping the assumption that the deficit adjust (via τ^c to match that path. The alternative assumption of an endogenous public debt and an exogenous tax is not practicable, as it would leads to explosive dynamics in most cases.

2.6 Wages

Wages are renegotiated in every period. They are determined by a standard Nash bargaining rule:

$$\max_{w_{a,t}} \left(\frac{\partial W^F_t}{\partial N_{a,t}} \right)^{1-\eta} \left(\frac{1}{u'_{c_{a,t}}} \frac{\partial W^H_t}{\partial N_{a,t}} \right)^{\eta}\,. \tag{24}$$

The first-order optimality condition can then be written:

$$(1-\eta)\, \frac{1}{u'_{c_{a,t}}} \frac{\partial W^H_t}{\partial N_{a,t}} = \eta\, \frac{1-\tau^w_t}{(1+\zeta_t)(1+\tau^c_t)} \frac{\partial W^F_t}{\partial N_{a,t}}\,. \tag{25}$$

[5] Changes in τ^c_t affect all incomes in the same way, while changes in τ^w_t for instance would change net replacement rates.

2.7 Equilibrium

Let Q_t denote the total financial value of firms at time t. In our deterministic setup, the return on equities must be equal to market interest rate. In other words, the value of equities must be such that, for all $t \geq 0$:

$$\frac{Q_{t+1} + \Pi_{t+1}}{Q_t} = R_{t+1} \,. \tag{26}$$

The left-hand side is the return of one unit of savings investment in equities while the right-hand side is the return if invested in firms' bonds.

Equilibrium on the capital market then implies:

$$K_{t+1} + Q_t = \sum_{a=0}^{14} s_{a,t}\, Z_{a,t} \,. \tag{27}$$

Let us now provide a formal definition of the equilibrium labor market frictions that we will next compare with an equilibrium with frictionless labor markets.

Definition 1 *Given the following exogenous processes and initial conditions:*

- *demographic variables* $\{x_t\}_{t=0..+\infty}$ *(fertility),* $\{\beta_{a,t}\}_{t=0..+\infty}^{a=0..15}$ *(mortality),* $\{X_{a,t}\}_{t=0..+\infty}^{a=0..15}$ *(migration) and* $\{z_{a,t}\}_{t=0..+\infty}^{a=0..7}$ *(participation),*
- *policy variables* $\{\rho_t^u, \rho_t^e, \rho_t^i\}_{t=0..+\infty}$ *(replacement rates) and* $\{\tau_t^w, \zeta_t\}_{t=0..+\infty}$ *(tax rates),*
- *initial population* $\{Z_{a,-1}\}^{a=0..15}$*, assets* $\{s_{a,-1}\}^{a=0..14}$ *and capital stock* $\bar{K}_0 < \sum_{a=0}^{14} s_{a,-1} Z_{a,-1}$*,*

an inter-temporal equilibrium with perfect foresight and labor market frictions *is such that:*

1. *saving* $\{s_{a,t}\}_{t=0..+\infty}^{a=0..14}$*, consumption* $\{c_{a,t}\}_{t=0..+\infty}^{a=0..15}$ *and retirement* $\{e_{a,t}, \lambda_{a,t}\}_{t=0..+\infty}^{a=6,7}$ *maximize households'utility (10) subject to budget constraint (11) and to (5),*

2. *capital input* $\{K_t\}_{t=0..+\infty}$*, posted vacancies* $\{V_t\}_{t=0..+\infty}$ *and output* $\{Y_t\}_{t=0..+\infty}$ *maximize firms'profits (16) subject to (8), (9), (14), and* $K_0 = \bar{K}_0$*,*

3. *the number of new hires* $\{M_t\}_{t=0..+\infty}$*, the probability of finding a job* $\{p_t\}_{t=0..+\infty}$ *and of filling a vacancy* $\{q_t\}_{t=0..+\infty}$*, and the employment rates* $\{n_{a,t}\}_{t=0..+\infty}^{a=0..7}$ *satisfy the matching technology (7), (8) and (9),*

4. *total and active population* $\{Z_{a,t}, P_{a,t}\}_{t=0..+\infty}^{a=0..7}$*, and number of job seekers* $\{\Omega_t\}_{t=0..+\infty}$ *satisfy the population dynamics (1), (2), (3) and (6).*

5. *unemployment* $\{u_{a,t}\}_{t=0..+\infty}^{a=0..7}$ *is such that the time constraint (4) holds.*

6. *wages* $\{w_{a,t}\}_{t=0..+\infty}^{a=0..7}$ *are negotiated following the Nash bargaining rule (24),*

7. *government benefits* $\{b_t^u\, b_t^e, b_t^i\}_{t=0..+\infty}$ *follow the rules defined by (20), and government spending* $\{G_t\}_{t=0..+\infty}$ *follow (22),*

8. *consumption taxes* $\{\tau_t^c\}_{t=0..+\infty}$ *are set by the government to balance its budget (23),*

9. *stock market prices and interest rate* $\{Q_t, R_t\}_{t=0..+\infty}$ *satisfy the arbitrage condition (26) and the financial market clearing condition (27).*

Definition 2 *Given the same exogenous processes and initial conditions as in Definition 1,* **an inter-temporal equilibrium with perfect foresight with frictionless labor market** *is such that:*

1. *saving* $\{s_{a,t}\}_{t=0..+\infty}^{a=0..14}$, *consumption* $\{c_{a,t}\}_{t=0..+\infty}^{a=0..15}$ *and retirement* $\{e_{a,t}, \lambda_{a,t}\}_{t=0..+\infty}^{a=6,7}$ *maximize households'utility (10) subject to budget constraint (11) and to (5),*

2. *capital input* $\{K_t\}_{t=0..+\infty}$, *posted vacancies* $\{V_t\}_{t=0..+\infty}$ *and output* $\{Y_t\}_{t=0..+\infty}$ *maximize firms'profits (16) subject to* $q_t = 1$, *(9), (14),* $a = 0$ *and* $K_0 = \bar{K}_0$,

3. *the number of new hires* $\{M_t\}_{t=0..+\infty}$ *satisfies* $M_t = V_t$, *the probability of finding a job* $\{p_t\}_{t=0..+\infty}$ *and of filling a vacancy* $\{q_t\}_{t=0..+\infty}$ *satisfy* $q_t = p_1 = 1$, *and the employment rates* $\{n_{a,t}\}_{t=0..+\infty,a=0..7}$ *satisfy (9),*

4. *total and active population* $\{Z_{a,t}, P_{a,t}\}_{t=0..+\infty}^{a=0..7}$, *and number of job seekers* $\{\Omega_t\}_{t=0..+\infty}$ *satisfy the population dynamics (1), (2), (3) and (6),*

5. *wages* $\{w_{a,t}\}_{t=0..+\infty}^{a=0..7}$ *are such that* $n_{a,t} + e_{a,t} = 1$,

6. *government benefits* $\{b_t^u\, b_t^e, b_t^i\}_{t=0..+\infty}$ *follow the rules defined by (20),*

7. *consumption taxes* $\{\tau_t^c\}_{t=0..+\infty}$ *are set by the government to balance its budget (23),*

8. *the interest rate* $\{R_t\}_{t=0..+\infty}$ *satisfies the financial market clearing condition (27).*

3 Calibration

Our objective is to assess the sustainability of public pension schemes, more specifically we want to evaluate the role of labor market imperfections and the interactions between pension and labor market reforms. Our calibration is meant to reproduce a European situation such as

France with fairly generous unemployment benefits and public pensions, with also significant early retirement compensations inducing low participation rates of elderly workers. The parameters of the model fall in three categories: (i) general technological and preference parameters; (ii) parameters specific to the frictional labor market; (iii) parameters determining taxes and transfers; (iv) parameters determining demographic changes (fertility, mortality, migration).

Technological and preference parameters

We assume a constant returns-to-scale Cobb-Douglas production function. The elasticity of output with respect to capital is set to $\alpha = 0.33$. In order to focus on the effects of demographics changes, we leave aside technological progress and assume constant values of the TFP and age-specific human capital parameters (A_t and $h_{a,t}$ respectively). Still, to reproduce the life cycle profile of wages, we assume that a worker's productivity increases with age till 50, and next decreases, but only very slowly so, as suggested by empirical findings. Bargained wages broadly follow the same pattern, with on top of the productivity effect a negative age effect due the shorter expected job duration of older workers.

We assume that utility is logarithmic in consumption, so that wealth and substitution effects of a change in the interest rate cancel each other. We set the labor disutility parameter d^n equal to 0.25 (which, at the steady state consumption level, represents about 15% of the wage income). There is no bequest motive. The subjective rate of time preference is given the customary value of 1% per quarter, implying (given the demographic structure) a steady state real interest rate of about 5.27% p.a. With these values, individual consumption rises over the life cycle and savings are negative during the first two periods of life; the capital-output ratio is equal to 2.50 (on a yearly basis). We fixed the leisure (early retirement) parameters at $d_6^e = 0.031$, $d_7^e = 0.049$ and $\phi = 0.20$, implying early retirement rates of 15 and 50% in the year 2000 (7.4 and 19% in the final steady state) for workers aged 50-59 and 60-64 respectively.

Labor market parameters

The matching function is specified as a constant returns-to-scale Cobb-Douglas with efficiency parameter μ. The elasticity of matches with respect to vacancies is fixed at $\nu = 0.50$. Similarly, the bargaining power of workers η is set at 0.5. The job separation rate χ is fixed at 2% per quarter. The vacancy cost a and the matching efficiency parameter μ are fixed at values such that, the steady state probabilities of filling a vacancy (over a five year period) and finding a job are both equal to 90%. The value of μ is 0.90; the vacancy cost ($\mathsf{a} = 43.5$) represents 33% of the gross wage of a 45 years old worker. With these parameter values, the steady state unemployment rates are equal to 10% for young workers (25-29), around 3.8% for adults (35-54), and around 3.3% for workers of retirement age.

Taxes and Transfers

Symbol	Value	Symbol	Value
Production function			
A	20.000	α	0.33
δ (quarterly)	0.025		
Age-dependent productivity			
h_0	2.70	h_1	3.30
h_2	3.85	h_3	4.45
h_4	5.00	h_5	5.80
h_6	5.60	h_7	5.50
Preferences			
β (quarterly)	0.990	d^n	0.250
d^e_6	0.031	d^e_7	0.049
ϕ	0.200		
Labor Market			
μ	0.90	ν	0.50
χ (quarterly)	0.02	a	43.50
η	0.50		
Policy variables			
ζ	0.35	τ^w	0.150
$\bar{g}$	0.20	ρ^u	0.400
ρ^e	0.50	ρ^i_t (year 2000)	0.550
ρ^i_t (year 2050)	0.35	ρ^i_t (year 2100)	0.325

Table 1: Parameter values

Government consumption is a constant fraction of GDP ($\bar{g}$ =20%). The gross replacement rate for unemployed workers is fixed at $\rho^u = 0.40$, implying a net replacement rate of about 47%. Actual net replacement rates vary a lot across countries and unemployment durations. The chosen value is close to the OECD median value for a two year unemployment spell (see OECD(2009), table 1.6). The reference wage used to compute pension benefits is typically an average over the best years of activity. For simplicity, we set the reference wage at the average wage of workers aged 45-64. At given replacement rate, our formulation implies that pensions are indexed on current wages (rather than prices). We calibrate the replacement rate ρ^i_t on French data, as reported in EC (2009). We set ρ^i at 0.55 in 2000. The French pension reforms of 1993 and 2003 are expected to progressively decrease this replacement rate through three main channels: (i) reference wage calculated over a larger period; (ii) indexation to prices rather than wages; (iii) increase in the number of years of activity required to have full pension rights. The effects of these reforms will appear progressively over the coming years and should imply by 2035 a 20 percentage points (30 percentage points by the year 2050) decrease in the value of the replacement rate defined above. The replacement for early retirement is assumed to remain constant, at $\rho^e = 0.50$.

Demographic variables

Expected population growth rates vary a lot across countries. We chose to calibrate the main demographic variables on French data. Survival probabilities from 1900 till 2100 are borrowed from Vallin-Meslé (2001). The expected decrease in mortality rates are illustrated in the left panel of figure 1. These changes imply that life expectancies at birth would increase from 77.9 years in 2000 to 86.5 in 2050 and 90.8 in 2095. Fertility and migration shocks (variables x_t and $X_{a,t+a}$) are calibrated so as to reproduce the population changes (by age category) observed or expected to take place from 1900 till 2100. The projections till 2050 are the latest ones published by INSEE (see Robert-Bobée (2006), updated April 2007)[6]. We assume that the population stabilizes progressively after 2050. These projections are illustrated in the right panel of figure 1. The reported population changes (total and adult -above 24-) imply a quasi constant population of age 25-64. The dependency ratio (defined as the ratio between population above 64 and population aged 25-64) has increased from about 10 percentage points from 1950 till 2000; it doubles during the next sixty years, to reach 60% in 2060 and stabilizes at 69% after 2100.

Once the model has been calibrated, we can assess the convergence speed by computing the eigenvalues of the Jacobian matrix of the linearized system around the steady state. It is generally found that such eigenvalues are complex numbers which generates non monotonic dynamics (see Azariadis, Bullard and Ohanian, 2004). Figure 2 represents the stable eigenvalues of the two models. Considering only the largest eigenvalues, which are those that matter as far

[6] Available at http://www.insee.fr/fr/themes/document.asp?ref_id=PROJPOPACT0650#a1

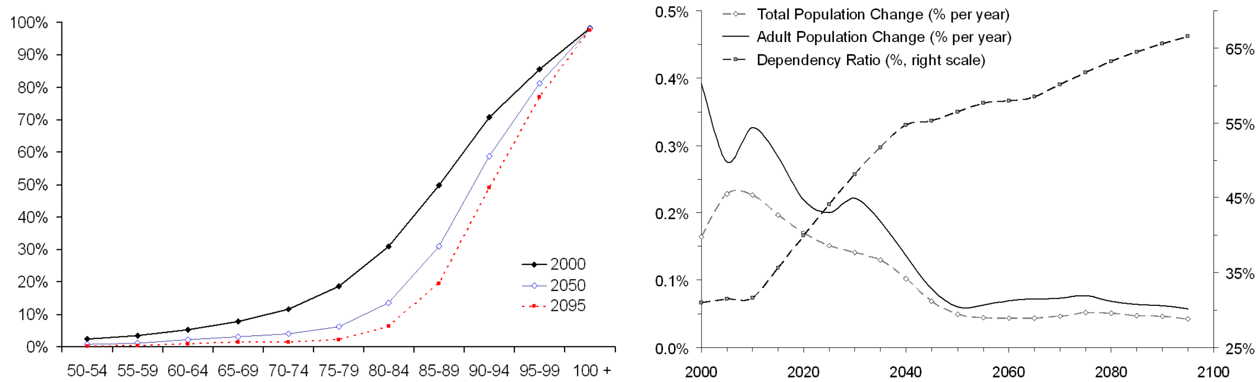

Figure 1: Projected age-specific death rates (left), (adult) population growth and dependency ratio (right)

as sluggishness is concerned, we observe that those of the model with frictions are larger, but the difference is very small. We conclude from this that labor market frictions do not mean more sluggishness compared to a Walrasian set-up, at least in an OLG model with five year long periods.

4 Simulation Results

We first present the simulation results obtained in the base scenario and illustrate the specific impact of migration and fertility shocks. We then highlight the role of labor market imperfections by contrasting the results of the base scenario with those obtained under the assumption of a competitive labor market. To illustrate the properties and quantitative implications of our model, we next examine two hypothetical policy scenarios assumed two take place after 2000, the elimination of early retirement compensations and the shift to a fully-funded pension scheme. We finally discuss the sensitivity of our results, especially about international capital flows.

4.1 Base Scenario

The base scenario incorporates the following exogenous shocks, described in the previous section: (i) decreasing mortality rates; (ii) fertility and migration shocks; (iii) decreasing replacement rate ρ_t^i after 2005, a consequence of the 1993 and 2003 pension reforms. We simulate the model under the assumption that the year 1900 was a steady state. After 2100, all exogenous variables keep the same value and the economy progressively converges to a new, final steady state. The

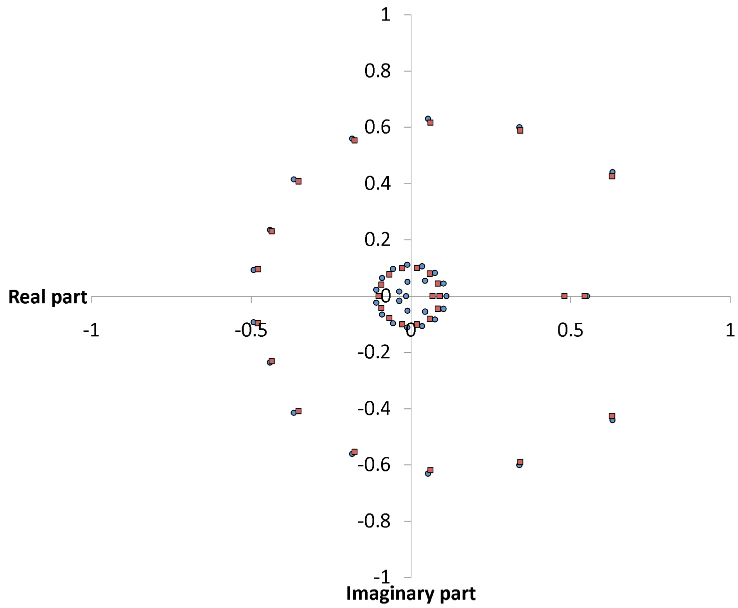

Figure 2: Eigenvalues of the model with frictions (circles) and without frictions (squares)

period of interest is the 21st century (2000-2100). Values simulated for that sub-period are unlikely to be affected by the initial conditions (1900 values).

Main Results

Figure 3 reproduces the evolution of some key variables in the base scenario. Population ageing implies increased savings and capital accumulation (not shown in the figure). Because of diminishing returns, the interest rate decreases. Capital accumulation also leads to a substantial increase in average labor productivity and wages, and also to an increase in the aggregate employment rate (about 6.4 percentage points). This higher employment rate comes from both a decrease in unemployment for young and mature workers (25-54) and from an increase in the participation rate of elderly workers (55-64). These effects are related to capital accumulation and lower interest rates, which increase profitability and the number of job openings. They imply a higher optimal average retirement age (less early retirement), at unchanged institutional environment. At given replacement rate, the higher participation rate of elderly workers is due mainly to the increased probability of having a job (see equation (12)), which we can interpret as a lower "discouragement" effect. The estimated cost of public pensions (including early retirement, and measured either in % of GDP) follows quite closely the projected values reported for France in in EC (2009), for the period 2000-2060, under the assumption of a (approximately) 5 percentage points (exogenous) increase in the aggregate employment rate. Thanks to the pension reforms decided in 1993/2003, the increase in public pension costs over

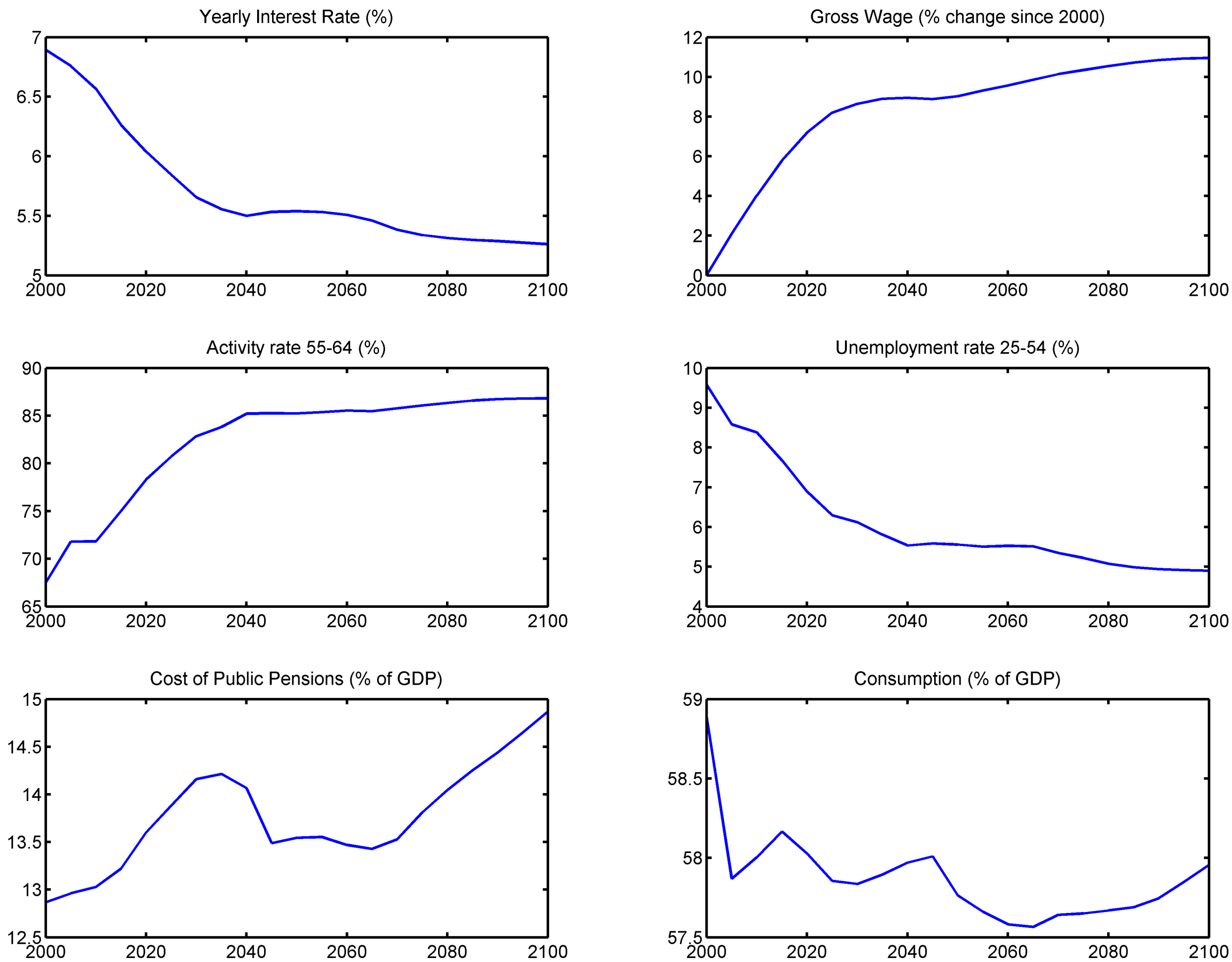

Figure 3: Base Scenario

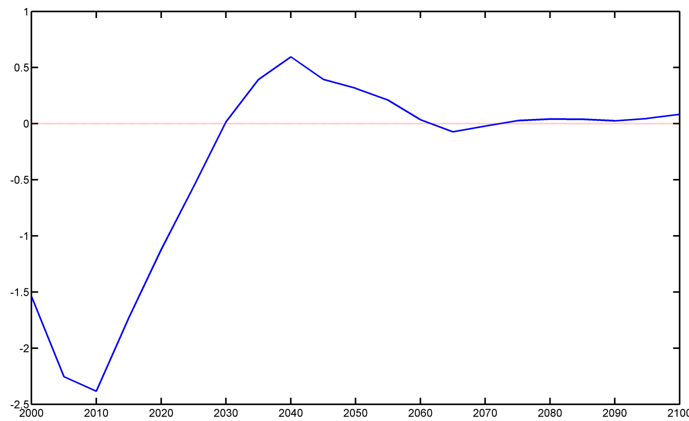

Figure 4: Contributions of fertility and migration shocks to the cost of public pensions (% of GDP)

the period 2000-2060 is no more than 1-1.5 percentage point (instead of 7 percentage points in a simulation without pension reform). We find however that, after 2070, the continuing ageing of the population may lead to additional and substantial increases in the cost of public pensions.

The contribution of fertility and migration shocks

At given pension scheme, the cost of public pensions in percent of GDP depends both on the expected increases in longevity and the size of the migration and fertility shocks. To separate these two effects, we simulated the model again with all fertility and migration variables set at their 2100 values. The results are summarized in Figure 4. The figure indicates by how much the cost of public pensions is changed by current and past fertility and migration shocks. During the first two decades, the effect is strongly negative. This is the continuation of the baby boom effect, which maintains the dependency ratio at low levels. After 2025, the effect becomes positive: baby-boomers retire, and the dependency ratio remains above normal for about five periods. After 2060, the effect of fertility and migration shocks becomes negligible, ie, the increase in the cost of public pensions after 2060 (see Figure 3) is solely driven by longevity effects.

4.2 Competitive vs Frictional Labor Markets

Increased longevity changes the savings behavior of households and pushes the equilibrium interest rate downwards. Firms use more capital, which pushes labor productivity and labor demand upwards. The final impact on employment depends on the working of the labor market. In models with search frictions, labor demand changes may affect the equilibrium

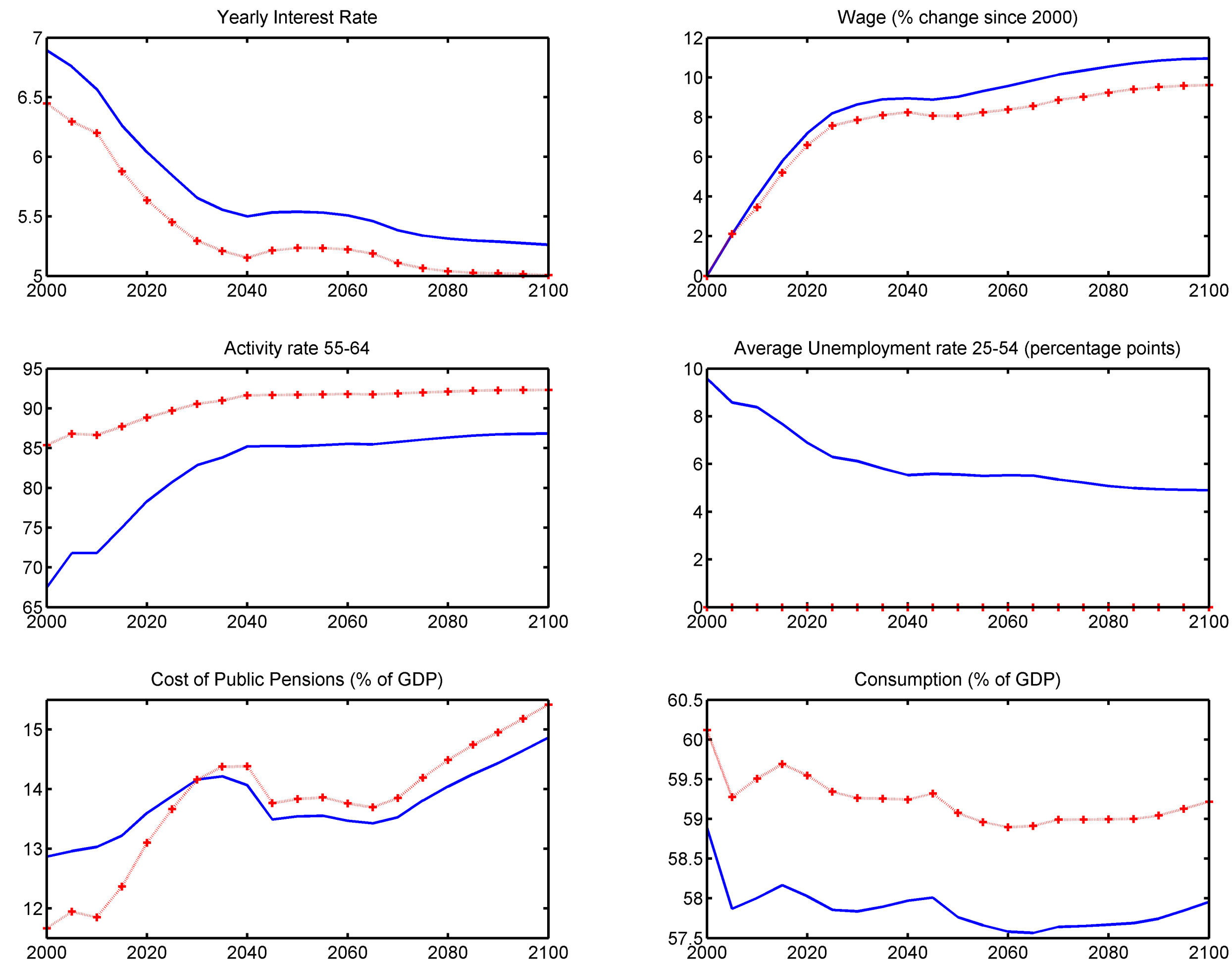

Figure 5: Ageing effects in a frictional (continuous line) vs perfectly competitive (dotted line with crosses) economy

(un-)employment rate. To evaluate the quantitative importance of such adjustments along the extensive margin, we simulate the equilibrium with frictionless labor market (Definition 2). The participation decision of elderly workers remains endogenous and determined in exactly the same fashion as before (see equation (12) for workers aged 60-64), where the employment probability for participating workers is now equal to one ($\pi_{7,t+7} = 1$).

The comparison between the two economies is summarized in Figure 5. Values for the frictional (resp. perfectly competitive) economy are represented by a continuous (resp. dotted with crosses) line. In the competitive setup, employment rate changes can only arise from changes in the participation rate of elderly workers. Because there is no unemployment risk and the market wage is higher, there is actually little early retirement in the competitive economy, although the generosity of the early retirement compensation is the same as in the frictional labor market economy. Despite increased labor productivity and wages, the participation rate of elderly workers thus barely changes and has a negligible impact on the aggregate employment rate. The comparison with the frictional labor market economy is striking. In the latter, the increased labor demand leads to higher employment probabilities for all age groups. A higher employment probability has a significant impact on the participation of elderly workers. Overall, the employment rate increases significantly, and stimulates further capital accumulation and investment. Altogether, the aggregate employment rate increases by more than 6%.

These differences explain why the two economies react differently to the pension reforms and the ensuing decrease in compensation rates. The cost of public pensions is initially smaller in the competitive economy (because there is less early retirement). It next increases by more than two percentage points and becomes larger than in the frictional economy. This difference reflects the different impact of pension reforms on employment rates, and the failure of the competitive model to account for adjustments along the extensive margin.

4.3 Policy scenarios

We focus on two policy scenarios and compare their implications in the frictional and the competitive economies. The first scenario is a complete elimination of all early retirement compensations after 2000. The second policy scenario is a shift to a fully-funded system, announced in 2005 and taking place in 2015. These two scenarios are admittedly extreme ones. They facilitate the comparisons between the two economies (frictional vs competitive) and the interpretation of the mechanisms at work.

Early retirement reform

The consequences of the elimination of all early retirement benefits are illustrated in Figure 6. This labor market policy change stimulates the participation of elderly workers, both in the frictional and in the competitive economy. All effects are qualitatively similar, but of a totally different magnitude, and much stronger in the frictional economy. In the competitive economy, because there is no unemployment and the equilibrium wage is higher, the initial participation rate is already high, and increases by 10 percentage points (from 85 to 95% at the time of the shock),against 25 percentage points (from 68 to 93%) in the frictional economy. The induced impact effects on employment, capital productivity (the interest rate), wages and consumption (both decline at impact) are thus much more pronounced in the frictional economy. There is also (at impact) an increase in the unemployment rate of younger workers. Eventually, the unemployment rate is back to its initial value, but employment, wages and output remain increase more in the frictional economy. The cost of public pensions (in % of GDP) decreases by 1.5 percentage points in the frictional economy (1 percentage point in the competitive economy).

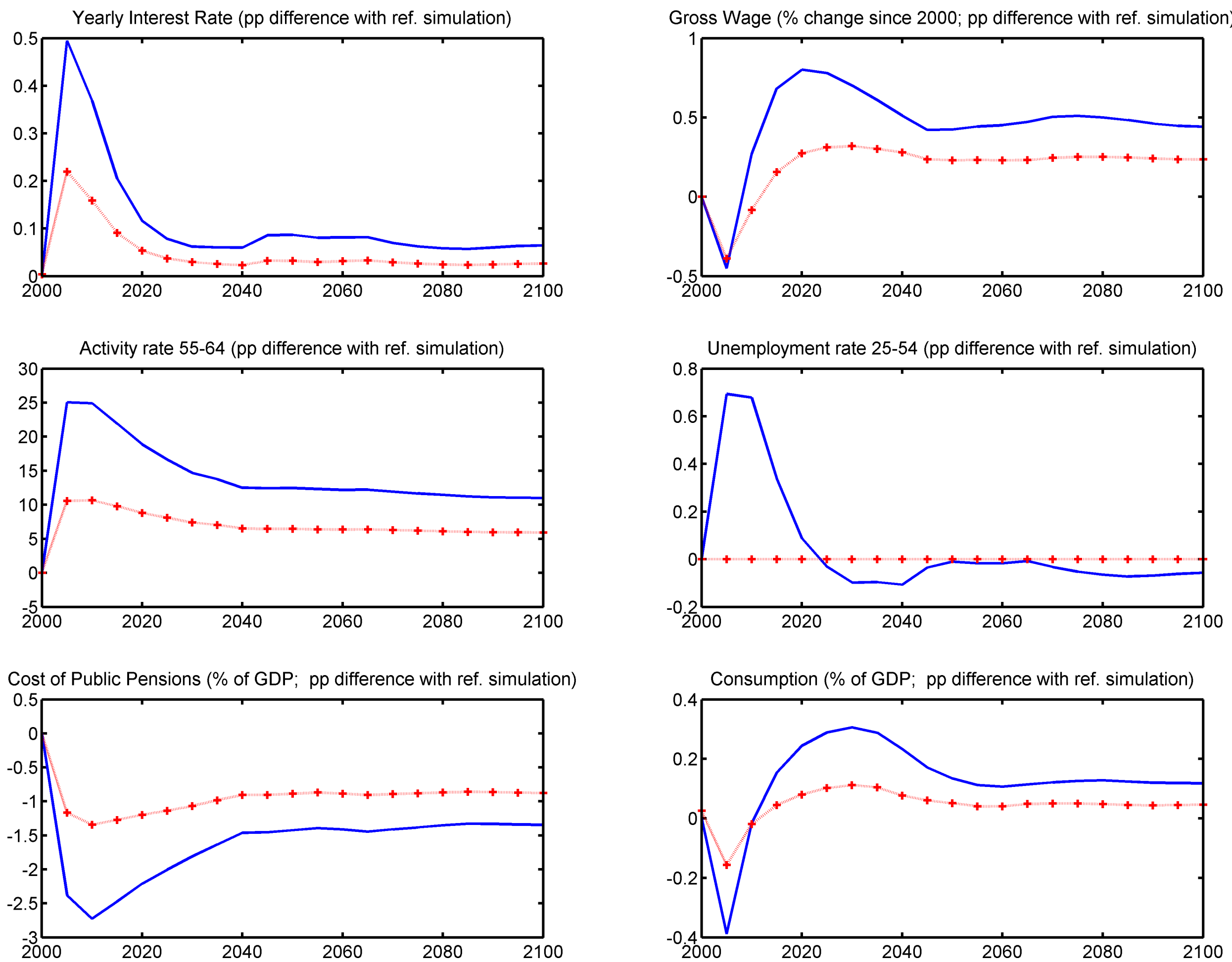

Figure 6: Consequences of eliminating early retirement benefits after 2000, respectively in the frictional (-) and the competitive economy (.+)

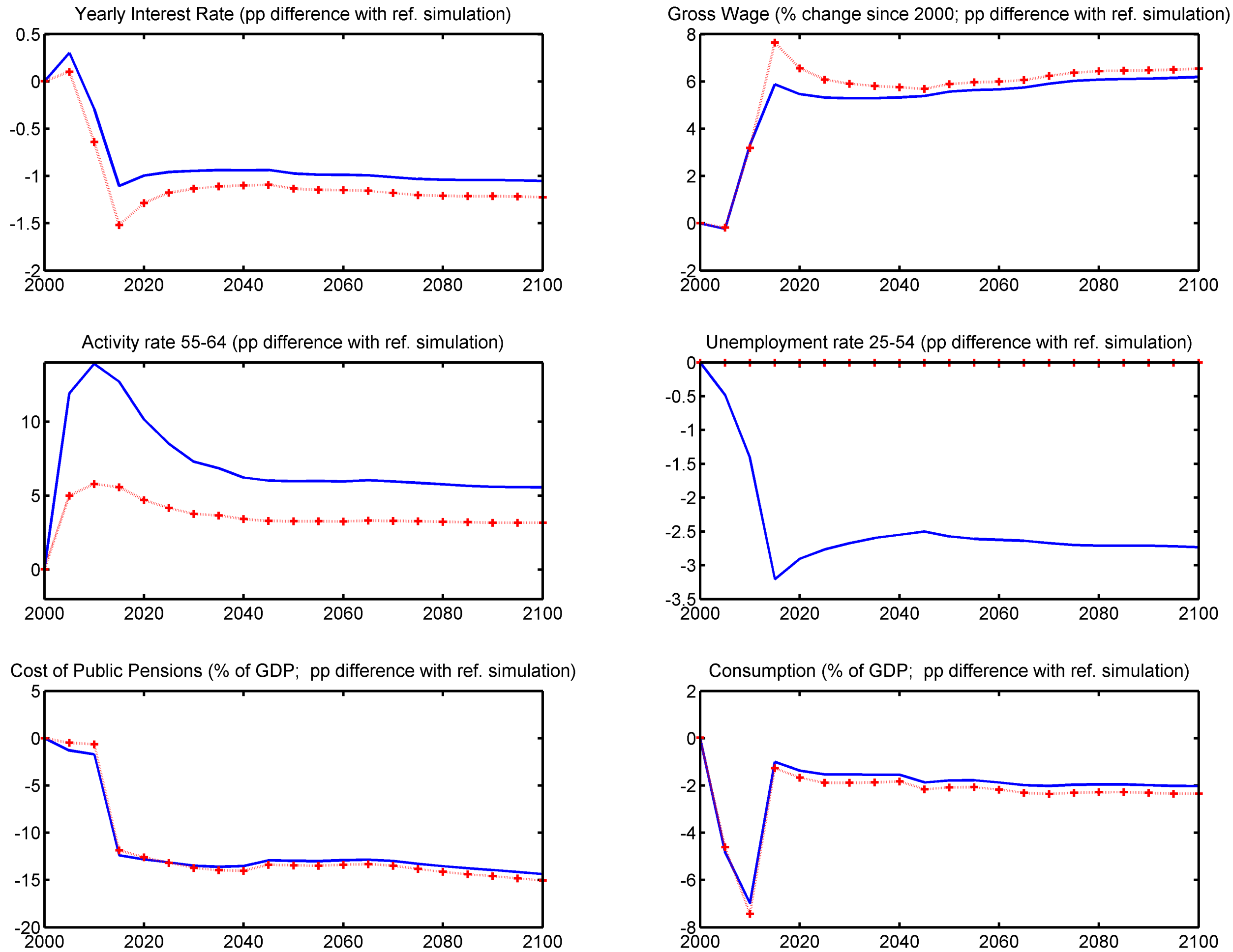

Figure 7: Consequences of announcing in 2005 a shift to a fully-funded system in 2015, respectively in the frictional (-) and the competitive economy (.+)

Shift from a PAYG to a FF pension scheme

Figure 7 illustrates the effects of announcing in 2005 a shift to a purely fully-funded system in 2015. We first consider the frictional economy. At impact, aggregate consumption falls, savings and capital accumulation rise, which leads (with a one-period lag) to a lower interest rate. The accumulation of capital and its positive effect on labor productivity stimulates labor demand and job creation (the unemployment rate starts falling) as well as wage growth. The activity rate of elderly workers increases, by 12 percentage points at impact (when consumption has fallen) and 5 percentage points in the long run (when consumption has increased). The equilibrium unemployment rate is about 2.5 percentage points lower in the long run, and implies a substantially larger output (and consumption) level.

In the competitive economy, the unemployment rate cannot vary; the participation rate of elderly workers increases by 3 percentage points. Aggregate employment barely changes. Capital accumulation leads to a larger output level, but this effects is not reinforced by job creation, as in the frictional economy.

4.4 Sensitivity Analysis: International Capital Flows

The objective of this section is to check to what extent our results might depend on specific assumptions. We repeated the previous simulation experiments for different values of a few key parameters: the subjective rate of time preference ($\beta = 1$ instead of 0.99), bargaining power ($\eta = 0.70$ instead of 0.50), concavity of leisure utility ($\phi = 0.1$ instead of 0.2). Such changes do not change our main conclusions and are not reproduced here. We rather focus on the role of international capital flows and their interactions with labor market imperfections. We do so by contrasting the closed economy scenario to the small open economy one. In the latter, the interest rate is exogenous. Most actual economies are in between these two extremes. Figure 8 reproduces the evolution of the key variables already considered in the base scenario (see figure 3) and makes the comparison with the evolution observed in the small open economy ("dashed +" line).

In both the closed and the small open economies, population ageing implies increased savings and asset accumulation. In the small open economy, agents can borrow or lend freely from the rest of the world, at given interest rate, assumed to remain constant. In a closed economy, agents can only save by accumulating domestic capital; because of diminishing returns, the interest rate decreases. This difference in interest rates explains all the other differences. Because in the small open economy returns to savings are constant, per capita consumption remains almost constant during the demographic transition, while it substantially decreases in the closed economy case. In the latter, domestic capital accumulation leads to a substantial increase in average labor productivity (hence in gross wages), and also an increase in aggregate employment. This higher employment level comes from both a decrease in the unemployment rate of young and mature workers (25-54) and from an increase in the participation rate of elderly workers (55-64). These effects are related to capital accumulation and lower interest rates, which increase profitability and the number of job openings. These changes imply a higher optimal average retirement age (less early retirement), at unchanged institutional environment. These effects are non-existent in the small open economy with fixed interest rates. This contrast has strong implications for the cost of public pensions. In the small open economy scenario, population ageing implies a 2 percentage point increase in the cost of public pensions between 2000 and 2060, compared to 0.5% in the closed economy. It is worth stressing that such a sharp difference is *not* observed in the perfect competition case. This is because in both the small open economy (with or without

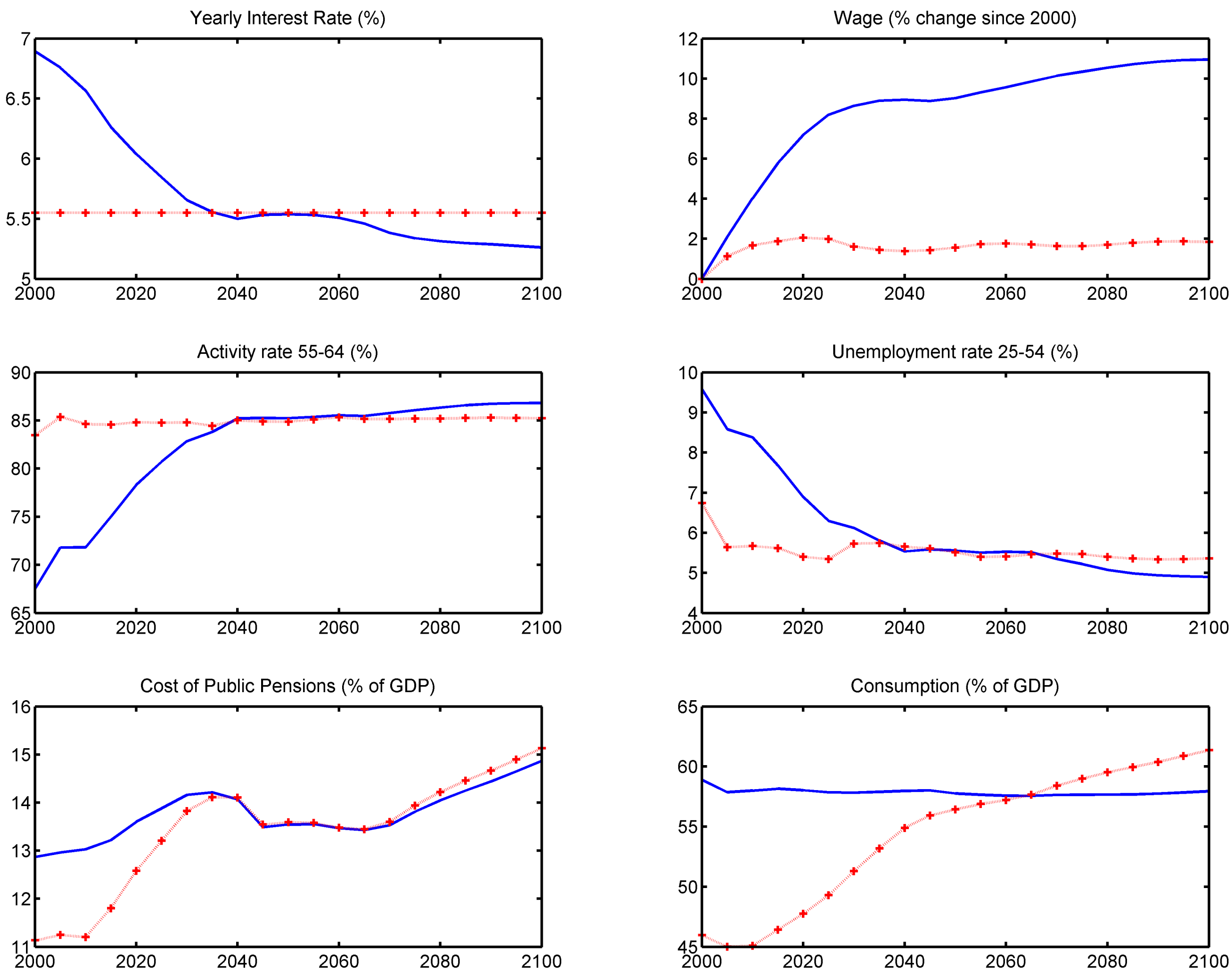

Figure 8: Comparing the closed (continuous line) and the small open economy (dotted line with crosses) scenarios

frictions) and in the competitive economy, the employment level does not vary much and does amplify the effects of policy changes.

Both Börsch-Supan et al. (2006) and Attanasio et al. (2007) have already stressed the role of international capital flows in determining how an economy will react to population ageing. We obtain a similar conclusion and emphasize that in economies with frictional labor market, changes in the equilibrium unemployment rate provide an additional mechanism by which international capital flows may affect the adjustment process and the future cost of public pensions (in % of GDP). With frictional labor markets, even the fiscal variables are affected by capital flows.

5 Conclusions

We have introduced labor frictions and early retirement decisions in an otherwise standard OLG setup. In this setup, we show that labor market imperfections interact strongly with policy variables. Social security changes have large effects on labor supply, at variance with results obtained in the equilibrium with a perfectly competitive labor market. In particular, generous early retirement benefits induce high early retirement rates only in the case where elderly workers face a risk of being unemployed.

This work is of course one step towards more realistic and useful representations of the labor market and its interactions with other variables in general equilibrium. The interest rate in particular plays a crucial role. One natural extension would be to use our model in a multi country setup (see for instance Docquier et al. (2010)), so as to have a better representation of the determinants of the equilibrium interest rate.

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Appendix: Household Optimization Problem

With initial and final financial wealth equal to zero (no bequests), the household's intertemporal budget constraint can be written as follows [7]:

$$\sum_{a=0}^{15} R_{t,t+a}^{-1}\,\beta_{a,t+a}\Bigg\{\Big[(1-\tau_{t+a}^{w})\,w_{a,t+a}\,n_{a,t+a}+b_{a,t+a}^{u}\,u_{a,t+a}+b_{a,t+a}^{e}\,e_{a,t+a} +b_{a,t+a}^{i}\,i_{a,t+a}\Big]\,.\,z_{a,t+a}-(1+\tau_{t+a}^{c})\,c_{a,t+a}\Bigg\}=0\,. \tag{28}$$

The discount factor $R_{t,t+a}$ is defined by $R_{t,t}=1$ and $R_{t,t+j}=\Pi_{j=1}^{a}\,R_{t+j}$ for $j\geq 1$.

The values of $c_{a,t+a}$, $\lambda_{6,t+6}$ and $\lambda_{7,t+7}$ maximizing the household objective function (10) subject to (5) and (9) and the intertemporal budget constraint (28) can thus be obtained from the maximization of the following Lagrangean function:

$$\frac{W_t^H}{Z_{0,t}}=\max_{c_{a,t+a},\,\lambda_{6,t+6},\,\lambda_{7,t+7}}\sum_{a=0}^{15}\beta_{a,t+a}\Bigg\{\beta^{a}\left(u(c_{a,t+a})-d^{n}\,n_{a,t+a}\,.\,z_{a,t+a}+d_{a}^{e}\,\frac{(e_{a,t+a})^{1-\phi}}{1-\phi}\,z_{a,t+a}\right) +\mu_t\,R_{t,t+a}^{-1}\left(\left[b_{a,t+a}^{u}+\Big((1-\tau_{t+a}^{w})\,w_{a,t+a}-b_{a,t+a}^{u}\Big)\,n_{a,t+a}+(b_{a,t+a}^{e}-b_{a,t+a}^{u})\,e_{a,t+a}\right].\,z_{a,t+a} -(1+\tau_{t+a}^{c})\,c_{a,t+a}\right)\Bigg\}$$

where μ_t is the Lagrange multiplier associated to the intertemporal budget constraint. The optimal values of $c_{a,t+a}$, $\lambda_{6,t+6}$ and $\lambda_{7,t+7}$ must satisfy the following first-order optimality conditions:

$$\beta^{a}\,\frac{u'_{c_{a,t+a}}}{1+\tau_{t+a}^{c}}=\mu_t\,R_{t,t+a}^{-1}\,; \tag{29}$$

$$\begin{aligned}&\left[d^{n}\,\frac{\partial n_{6,t+6}}{\partial\lambda_{6,t+6}}-d_{6}^{e}\,(e_{6,t+6})^{-\phi}\,\frac{\partial e_{6,t+6}}{\partial\lambda_{6,t+6}}\right]+\beta\,\frac{\beta_{7,t+7}}{\beta_{6,t+6}}\left[d^{n}\,\frac{\partial n_{7,t+7}}{\partial\lambda_{6,t+6}}-d_{7}^{e}\,(e_{7,t+7})^{-\phi}\,\frac{\partial e_{7,t+7}}{\partial\lambda_{6,t+6}}\right]\\&=\frac{u'_{c_{6,t+6}}}{1+\tau_{t+6}^{c}}\left[\Big((1-\tau_{t+6}^{w})\,w_{6,t+6}-b_{6,t+6}^{u}\Big)\,\frac{\partial n_{6,t+6}}{\partial\lambda_{6,t+6}}+\Big(b_{6,t+6}^{e}-b_{6,t+6}^{u}\Big)\,\frac{\partial e_{6,t+6}}{\partial\lambda_{6,t+6}}\right]\\&+\beta\,\frac{\beta_{7,t+7}}{\beta_{6,t+6}}\,\frac{u'_{c_{7,t+7}}}{1+\tau_{t+7}^{c}}\left[\Big((1-\tau_{t+7}^{w})\,w_{7,t+7}-b_{7,t+7}^{u}\Big)\,\frac{\partial n_{7,t+7}}{\partial\lambda_{6,t+6}}+\Big(b_{7,t+7}^{e}-b_{7,t+7}^{u}\Big)\,\frac{\partial e_{7,t+7}}{\partial\lambda_{6,t+6}}\right];\end{aligned} \tag{30}$$

$$\begin{aligned}&\left[d^{n}\,\frac{\partial n_{7,t+7}}{\partial\lambda_{7,t+7}}-d_{7}^{e}\,(e_{7,t+7})^{-\phi}\,\frac{\partial e_{7,t+7}}{\partial\lambda_{7,t+7}}\right]\\&=\frac{u'_{c_{7,t+7}}}{1+\tau_{t+7}^{c}}\left[\Big((1-\tau_{t+7}^{w})\,w_{7,t+7}-b_{7,t+7}^{u}\Big)\,\frac{\partial n_{7,t+7}}{\partial\lambda_{7,t+7}}+\Big(b_{7,t+7}^{e}-b_{7,t+7}^{u}\Big)\,\frac{\partial e_{7,t+7}}{\partial\lambda_{7,t+7}}\right].\end{aligned} \tag{31}$$

[7] It is clear from equation (28) that consumption taxes have the same effect as income taxes.

In these expressions,

$$\frac{\partial e_{6,t+6}}{\partial \lambda_{6,t+6}} = 1\,; \qquad \frac{\partial e_{7,t+7}}{\partial \lambda_{6,t+6}} = (1-\lambda_{7,t+7})\,; \qquad \frac{\partial e_{7,t+7}}{\partial \lambda_{7,t+7}} = (1-\lambda_{6,t+6})\,;$$
$$\frac{\partial n_{6,t+6}}{\partial \lambda_{6,t+6}} = -\frac{n_{t+6}}{1-\lambda_{t+6}}\,; \qquad \frac{\partial n_{7,t+7}}{\partial \lambda_{6,t+6}} = -\frac{n_{t+7}}{1-\lambda_{t+6}}\,; \qquad \frac{\partial n_{7,t+7}}{\partial \lambda_{7,t+7}} = -\frac{n_{t+7}}{1-\lambda_{t+7}}\,.$$

The first optimality condition (29) is the usual Euler condition. It implies:

$$\frac{u'_{c_{a,t+a}}}{1+\tau^c_{t+a}} = \beta\, R_{t+a+1}\, \frac{u'_{c_{a+1,t+a+1}}}{1+\tau^c_{t+a+1}}\,.$$

The other two optimality conditions are specific to this model and determine the activity rate of senior workers. After substitution and rearrangements and with the assumption that $u(c_{a,t+a})$ is logarithmic, these optimality conditions can be recast as follows:

$$\begin{aligned}
&\left[\frac{b^e_{6,t+6}-b^u_{6,t+6}}{(1+\tau^c_{t+6})\,c_{6,t+6}} + d^e_6\,(e_{6,t+6})^{-\phi}\right](1-e_{6,t+6}) \\
&\quad + \beta\,\frac{\beta_{7,t+7}}{\beta_{6,t+6}}\left[\frac{b^e_{7,t+7}-b^u_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}} + d^e_7\,(e_{7,t+7})^{-\phi}\right](1-e_{7,t+7}) \\
&= \left[\frac{(1-\tau^w_{t+6})\,w_{6,t+6}-b^u_{6,t+6}}{(1+\tau^c_{t+6})\,c_{6,t+6}} - d^n\right] n_{6,t+6} \\
&\quad + \beta\,\frac{\beta_{7,t+7}}{\beta_{6,t+6}}\left[\frac{(1-\tau^w_{t+7})\,w_{7,t+7}-b^u_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}} - d^n\right] n_{7,t+7}\,,
\end{aligned} \tag{32}$$

and

$$\left[\frac{b^e_{7,t+7}-b^u_{7,t+7}}{(1+\tau^c_{t+7})c_{7,t+7}} + d^e_7\,(e_{7,t+7})^{-\phi}\right](1-e_{7,t+7}) = \left[\frac{(1-\tau^w_{t+7})w_{7,t+7}-b^u_{7,t+7}}{(1+\tau^c_{t+7})c_{7,t+7}} - d^n\right] n_{7,t+7} \tag{33}$$

The economic interpretation of these optimality conditions becomes easier if we notice that the unconditional probability of having a job is given by:

$$\pi_{a,t+a} = \frac{n_{a,t+a}}{n_{a,t+a}+u_{a,t+a}} = \frac{n_{a,t+a}}{1-e_{a,t+a}}\,,$$

so that the last optimality condition for instance can be written as follows:

$$\frac{b^e_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}} + d^e_7\,(e_{7,t+7})^{-\phi} = \pi_{7,t+7}\left[\frac{(1-\tau^w_{t+7})\,w_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}} - d^n\right] + (1-\pi_{7,t+7})\left[\frac{b^u_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}}\right],$$

and similarly for the other optimality condition.