1 Derivation of recursive auxiliary variables from a Calvo price setup.

The first order condition of the price setting problem is as follows:

$$\sum_{k=0}^{\infty}\theta_P^k\mathbb{E}_t\left[\Lambda_{t,t+k}Y_{t+k}^*\left(z\right)\frac{1}{P_{t+k}}\left(P_t^*\left(z\right)-\left(\frac{\varepsilon_R}{\varepsilon_R-1}\right)P_{t+k}^{int}\right)\right]=0$$

Facing the individual demand curve:

$$Y_t\left(z\right)=\left(\frac{P_t\left(z\right)}{P_t}\right)^{-\varepsilon_R}Y_t$$

Inserting the demand corresponding the optimal price, P^*(dropping the z index, as a symmetric equilibrium is assumed):

$$\sum_{k=0}^{\infty}\theta_P^k\mathbb{E}_t\left[\Lambda_{t,t+k}\left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_R}Y_{t+k}\frac{1}{P_{t+k}}\left(P_t^*-\left(\frac{\varepsilon_R}{\varepsilon_R-1}\right)P_{t+k}^{int}\right)\right]=0$$

$$\Rightarrow$$

$$\sum_{k=0}^{\infty}\theta_P^k\mathbb{E}_t\left[\Lambda_{t,t+k}\left(\frac{P_t}{P_{t+k}}\right)^{-\varepsilon_R}\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon_R}Y_{t+k}\frac{P_t}{P_{t+k}}\left(\frac{P_t^*}{P_t}-\left(\frac{\varepsilon_R}{\varepsilon_R-1}\right)\frac{P_{t+k}^{int}}{P_t}\right)\right]=0$$

Inserting for the stochastic discount factor:

$$\sum_{k=0}^{\infty}\theta_P^k\mathbb{E}_t\left[\beta_P^k\frac{C_{P,t}}{C_{P,t+k}}\left(\frac{P_t}{P_{t+k}}\right)^{-\varepsilon_R}\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon_R}Y_{t+k}\frac{P_t}{P_{t+k}}\left(\frac{P_t^*}{P_t}-\left(\frac{\varepsilon_R}{\varepsilon_R-1}\right)\frac{P_{t+k}^{int}}{P_t}\right)\right]=0$$

Dividing through with $C_{P,t}$ and using $\pi_t^*=\frac{P_t^*}{P_t}$ to collect terms:

$$\sum_{k=0}^{\infty}\theta_P^k\mathbb{E}_t\left[\beta_P^k\frac{1}{C_{P,t+k}}\left(\frac{P_t}{P_{t+k}}\right)^{1-\varepsilon_R}\left(\pi_t^*\right)^{1-\varepsilon_R}Y_{t+k}\right]=\sum_{k=0}^{\infty}\theta_P^k\mathbb{E}_t\left[\beta_P^k\frac{1}{C_{P,t+k}}\left(\frac{P_t}{P_{t+k}}\right)^{1-\varepsilon_R}\left(\pi_t^*\right)^{-\varepsilon_R}Y_{t+k}\left(\frac{\varepsilon_R}{\varepsilon_R-1}\right)\frac{P_{t+k}^{int}}{P_t}\right]$$

Collecting π_t^* on left hand side, and factoring out:

$$\sum_{k=0}^{\infty}\theta_P^k\mathbb{E}_t\left[\beta_P^k\frac{1}{C_{P,t+k}}\left(\frac{P_t}{P_{t+k}}\right)^{1-\varepsilon_R}\pi_t^*Y_{t+k}\right]=\sum_{k=0}^{\infty}\theta_P^k\mathbb{E}_t\left[\beta_P^k\frac{1}{C_{P,t+k}}\left(\frac{P_t}{P_{t+k}}\right)^{1-\varepsilon_R}Y_{t+k}\left(\frac{\varepsilon_R}{\varepsilon_R-1}\right)\frac{P_{t+k}^{int}}{P_t}\right]$$

$$\Rightarrow$$

$$\pi_t^*\sum_{k=0}^{\infty}\theta_P^k\mathbb{E}_t\left[\frac{\beta_P^k}{C_{P,t+k}}\left(\frac{P_t}{P_{t+k}}\right)^{1-\varepsilon_R}Y_{t+k}\right]=\left(\frac{\varepsilon_R}{\varepsilon_R-1}\right)\sum_{k=0}^{\infty}\theta_P^k\mathbb{E}_t\left[\frac{\beta_P^k}{C_{P,t+k}}\left(\frac{P_t}{P_{t+k}}\right)^{1-\varepsilon_R}Y_{t+k}\frac{P_{t+k}^{int}}{P_t}\right]$$

Defining two auxiliary variables, this equation can be expressed as:

$$\pi_t^*q_{1,t}=\left(\frac{\varepsilon_R}{\varepsilon_R-1}\right)q_{2,t}$$

where:

$$
\begin{aligned}
q_{1,t} =& \sum_{k=0}^{\infty} \theta_P^k \mathbb{E}_t \left[\frac{\beta_P^k}{C_{P,t+k}} \left(\frac{P_t}{P_{t+k}} \right)^{1-\varepsilon_R} Y_{t+k} \right] \\
=& \frac{1}{C_{P,t}} Y_t + \sum_{k=1}^{\infty} \theta_P^k \mathbb{E}_t \left[\frac{\beta_P^k}{C_{P,t+k}} \left(\frac{P_t}{P_{t+k}} \right)^{1-\varepsilon_R} Y_{t+k} \right] \\
=& \frac{1}{C_{P,t}} Y_t + \mathbb{E}_t \left[\left(\frac{P_t}{P_{t+1}} \right)^{1-\varepsilon_R} \right] \sum_{k=1}^{\infty} \theta_P^k \mathbb{E}_t \left[\frac{\beta_P^k}{C_{P,t+k}} \left(\frac{P_{t+1}}{P_{t+k}} \right)^{1-\varepsilon_R} Y_{t+k} \right] \\
=& \frac{1}{C_{P,t}} Y_t + \beta_P \theta_P \mathbb{E}_t \left[(\pi_{t+1})^{\varepsilon_R - 1} q_{1,t+1} \right]
\end{aligned}
$$

and:

$$
\begin{aligned}
q_{2,t} =& \sum_{k=0}^{\infty} \theta_P^k \mathbb{E}_t \left[\frac{\beta_P^k}{C_{P,t+k}} \left(\frac{P_t}{P_{t+k}} \right)^{1-\varepsilon_R} Y_{t+k} \frac{P_{t+k}^{int}}{P_t} \right] \\
=& \frac{1}{C_{P,t}} Y_t \frac{P_t^{int}}{P_t} + \sum_{k=1}^{\infty} \theta_P^k \mathbb{E}_t \left[\frac{\beta_P^k}{C_{P,t+k}} \left(\frac{P_t}{P_{t+k}} \right)^{1-\varepsilon_R} Y_{t+k} \frac{P_{t+k}^{int}}{P_t} \right] \\
=& \frac{1}{C_{P,t}} Y_t \frac{P_t^{int}}{P_t} + \mathbb{E}_t \left[\left(\frac{P_t}{P_{t+1}} \right)^{1-\varepsilon_R} \right] \sum_{k=1}^{\infty} \theta_P^k \mathbb{E}_t \left[\frac{\beta_P^k}{C_{P,t+k}} \left(\frac{P_{t+1}}{P_{t+k}} \right)^{1-\varepsilon_R} Y_{t+k} \frac{P_{t+k}^{int}}{P_t} \right] \\
=& \frac{1}{C_{P,t}} Y_t \frac{P_t^{int}}{P_t} + \beta_p \theta_P \mathbb{E}_t \left[(\pi_{t+1})^{\varepsilon_R - 1} q_{2,t+1} \right]
\end{aligned}
$$

Furthermore, we can rearrange the aggregate price evolution, such that we get a system only in inflation, instead of price levels:

$$
\begin{aligned}
P_t &= \left(\theta_P P_{t-1}^{1-\varepsilon_R} + (1-\theta_P) \left(P_t^* \right)^{1-\varepsilon_R} \right)^{\frac{1}{1-\varepsilon_R}} \\
&\Rightarrow \\
P_t^{1-\varepsilon_R} &= \theta_P P_{t-1}^{1-\varepsilon_R} + (1-\theta_P) \left(P_t^* \right)^{1-\varepsilon_R} \\
&\Rightarrow \\
1 &= \theta_P \frac{P_{t-1}^{1-\varepsilon_R}}{P_t^{1-\varepsilon_R}} + (1-\theta_P) \frac{\left(P_t^* \right)^{1-\varepsilon_R}}{P_t^{1-\varepsilon_R}} \\
&= \theta_P \left(\frac{P_t}{P_{t-1}} \right)^{\varepsilon_R - 1} + (1-\theta_P) \left(\frac{P_t^*}{P_t} \right)^{1-\varepsilon_R} \\
&= \theta_P \left(\pi_t \right)^{\varepsilon_R - 1} + (1-\theta_P) \left(\pi_t^* \right)^{1-\varepsilon_R}
\end{aligned}
$$