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Technical Appendix to

THE MACROECONOMIC EFFECTS OF LARGE-SCALE ASSET PURCHASE PROGRAMMES*

Han Chen, Vasco Cúrdia and Andrea Ferrero

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Appendix A. Model

A.1. *Final Goods Producers*

The final good Y_t is a composite made of a continuum of goods indexed by $i \in (0,1)$

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} \mathrm{d}i\right]^{1+\lambda_f}. \tag{A.1}$$

The **final goods producers** buy the intermediate goods on the market, package Y_t, and resell it to consumers. These firms maximise profits in a perfectly competitive environment. Their problem is

$$\begin{aligned} &\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) \mathrm{d}i \\ &\text{s.t.}\, Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} \mathrm{d}i\right]^{1+\lambda_f} (\mu_{f,t}). \end{aligned} \tag{A.2}$$

The first order conditions (FOCs) are

$$[\partial Y_t] : P_t = \mu_{f,t}. \tag{A.3}$$

$$[\partial Y_t(i)] : -P_t(i) + \mu_{f,t}[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} \mathrm{d}i]^{\lambda_f} Y_t(i)^{-\frac{\lambda_f}{1+\lambda_f}} = 0. \tag{A.4}$$

Note that

$$\left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} \mathrm{d}i\right]^{\lambda_f} = Y_t^{\frac{\lambda_f}{1+\lambda_f}}.$$

From the FOCs one obtains

* The content of this paper does not necessarily reflect the views of the Federal Reserve Bank of New York or the Federal Reserve System.

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_t.$$

Combining this condition with the zero-profit condition (because these firms operate in a perfectly competitive market), one obtains the expression for the price of the composite good:

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_f}} \mathrm{d}i\right]^{-\lambda_f}. \tag{A.5}$$

Note that the elasticity is $(1 + \lambda_f)/\lambda_f$. $\lambda_f = 0$ corresponds to the linear case. $\lambda_f \to \infty$ corresponds to the Cobb–Douglas case. We will constrain $\lambda_f \in (0,\infty)$.

A.2. *Intermediate Goods Producers*

Intermediate goods producer i uses the following technology:

$$Y_t(i) = Z_t^{1-\alpha} K_t(i)^\alpha L_t(i)^{1-\alpha}. \tag{A.6}$$

The log of the growth rate of productivity $z_t = \log\left(\frac{Z_t/Z_{t-1}}{1+\gamma}\right)$ follows the process

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t}, \epsilon_{z,t} \sim \mathbb{N}(0, \sigma^2_{\varepsilon_z}). \tag{A.7}$$

The firm's profit is given by

$$P_t(i)Y_t(i) - W_t L_t(i) - R_t^k K_t(i).$$

Cost minimisation subject to (A.6) yields the conditions:

$$\begin{aligned}
[\partial L_t(i)] &: \mathcal{V}_t(i)(1-\alpha) Z_t^{1-\alpha} K_t(i)^\alpha L_t(i)^{-\alpha} = W_t \\
[\partial K_t(i)] &: \mathcal{V}_t(i)\alpha Z_t^{1-\alpha} K_t(i)^{\alpha-1} L_t(i)^{1-\alpha} = R_t^k,
\end{aligned}$$

where $\mathcal{V}_t(i)$ is the Lagrange multiplier associated with (A.6). In turn, these conditions imply:

$$\frac{K_t(i)}{L_t(i)} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}.$$

Note that if we integrate both sides of the equation with respect to i and define $K_t = \int K_t(i)\mathrm{d}i$ and $L_t = \int L_t(i)di$, we obtain a relationship between aggregate labour and capital:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k} L_t. \tag{A.8}$$

The marginal cost MC_t is the same for all firms and equal to:

$$\begin{aligned}
MC_t &= \left[W_t + R_t^k \frac{K_t(i)}{L_t(i)}\right] Z_t^{-(1-\alpha)} \left(\frac{K_t(i)}{L_t(i)}\right)^{-\alpha} \\
&= \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} (R_t^k)^\alpha [(1+\gamma)\mathrm{e}^{z_t}]^{-(1-\alpha)}.
\end{aligned} \tag{A.9}$$

Profits can then be expressed as $[P_t(i) - \lambda_{f,t} MC_t] Y_t(i)$, where $\lambda_{f,t}$ is a shock to the time-varying price markup, assumed to follow the exogenous process:

$$\ln\lambda_{f,t} = \rho_{\lambda_f} \ln\lambda_{f,t-1} + \epsilon_{\lambda,t}, \epsilon_{\lambda,t} \sim \mathbb{N}(0, \sigma^2_{\varepsilon_\lambda}). \tag{A.10}$$

Prices are sticky as in Calvo (1983). Specifically, each firm can readjust prices with probability $1 - \zeta_p$ in each period. We depart from Calvo (1983) in assuming that for those firms that cannot adjust prices, $P_t(i)$ will increase at the steady-state rate of inflation π. For those firms that can adjust prices, the problem is to choose a price level $\tilde{P}_t(i)$ that maximises the expected present discounted value of profits in all states of nature where the firm is stuck with that price in the future:

$$\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Xi_{t+s}^p [\tilde{P}_t(i)\Pi^s - \lambda_{f,t+s} MC_{t+s}] Y_{t+s}(i)$$
$$\text{s.t. } Y_{t+s}(i) = \left[\frac{\tilde{P}_t(i)\Pi^s}{P_{t+s}}\right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_{t+s}, \tag{A.11}$$

where $\Pi \equiv 1 + \pi$, and Ξ_{t+s}^p is today's value of a future dollar for the average shareholder. This variable is the appropriate discount factor of future dividends because we assume that ownership of intermediate goods producing firms is equally distributed among all households. The definition of average marginal utility is

$$\Xi_{t+s}^p \equiv \sum_j \omega_j \beta_j^s \Xi_{t+s}^{j,p},$$

where ω_j represents the measure of type j in the population.

The FOC for the firm is

$$0 = \tilde{P}_t(i)\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Xi_{t+s}^p \frac{1}{\lambda_f} \Pi^{s\left(1-\frac{1+\lambda_f}{\lambda_f}\right)} P_{t+s}^{\frac{1+\lambda_f}{\lambda_f}} Y_{t+s}$$
$$- \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Xi_{t+s}^p \frac{1+\lambda_f}{\lambda_f} \Pi^{-s\frac{1+\lambda_f}{\lambda_f}} P_{t+s}^{\frac{1+\lambda_f}{\lambda_f}} Y_{t+s} \lambda_{f,t+s} MC_{t+s}. \tag{A.12}$$

Note that all firms readjusting prices face an identical problem. We will consider only the symmetric equilibrium in which all firms that can readjust prices will choose the same $\tilde{P}_t(i)$, so we can drop the i index from now on. From (A.5) it follows that

$$P_t = \left[(1-\zeta_p)\tilde{P}_t^{-\frac{1}{\lambda_f}} + \zeta_p(\Pi P_{t-1})^{-\frac{1}{\lambda_f}}\right]^{-\lambda_f}. \tag{A.13}$$

A.3. *Capital Producers*

There is a representative firm, owned by all households, that operates under perfect competition, invests in capital, chooses utilisation and rents it to intermediate firms. By choosing the utilisation rate u_t, capital producers end up renting in each period t an amount of 'effective' capital equal to

$$K_t = u_t \bar{K}_{t-1}, \tag{A.14}$$

where R_t^k is the return per unit of effective capital. Utilisation, however, subtracts real resources measured in terms of the consumption good

$$a(u_t)\bar{K}_{t-1}.$$

The law of motion of capital is

$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + \mu_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{A.15}$$

where $\delta \in (0,1)$ is the depreciation rate and $S(\cdot)$ is the cost of adjusting investment, with $S'(\cdot) > 0$ and $S''(\cdot) > 0$.

Capital producers maximise expected discounted stream of dividends to their shareholders:

$$\max_{\bar{K}_t, u_t, I_t} \mathbb{E}_t \sum_{s=0}^{\infty} (\omega_u \beta_u^s \Xi_{t+s}^{p,u} + \omega_r \beta_r^s \Xi_{t+s}^{p,r})[R_{t+s}^k u_{t+s} \bar{K}_{t+s-1} - P_{t+s} a(u_{t+s}) \bar{K}_{t+s-1} - P_{t+s} I_{t+s}]$$

subject to the law of motion (LOM) of capital (A.15), with Q_t the Lagrange multiplier associated with the constraint, and consider that the multiplier for time $t+s$ constraint is premultiplied by $(\omega_u \beta_u^s \Xi_{t+s}^{p,u} + \omega_r \beta_r^s \Xi_{t+s}^{p,r})$. FOC are:

$$[\partial u_t] : 0 = R_t^k - P_t a'(u_t), \tag{A.16}$$

$$[\partial \bar{K}_t] : 0 = \mathbb{E}_t \left\{ \frac{\omega_u \beta_u \Xi_{t+1}^{p,u} + \omega_r \beta_r \Xi_{t+1}^{p,r}}{\omega_u \Xi_t^{p,u} + \omega_r \Xi_t^{p,r}} [R_{t+1}^k u_{t+1} - P_{t+1} a(u_{t+1}) + (1-\delta) Q_{t+1}] \right\} - Q_t, \tag{A.17}$$

$$\begin{aligned} [\partial I_t] : 0 = & -1 + \frac{Q_t}{P_t} \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] - \frac{Q_t}{P_t} \mu_t S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \\ & + \mathbb{E}_t \left[\frac{(\omega_u \beta_u \Xi_{t+1}^{p,u} + \omega_r \beta_r \Xi_{t+1}^{p,r}) P_{t+1}}{(\omega_u \Xi_t^{p,u} + \omega_r \Xi_t^{p,r}) P_t} \frac{Q_{t+1}}{P_{t+1}} \mu_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]. \end{aligned} \tag{A.18}$$

A.4. *Households*

The key modification relative to the standard model is the introduction of long-term bonds and segmentation. We follow the formulation in Woodford (2001) and consider long-term bonds with coupon equal to κ^s paid at time $t+1+s$, for $s \geq 0$. This implies that the gross yield to maturity is given by

$$R_{L,t} = \frac{1}{P_{L,t}} + \kappa, \tag{A.19}$$

or, equivalently, the price of such bond is given by

$$P_{L,t} = \frac{1}{R_{L,t} - \kappa}. \tag{A.20}$$

The duration of this bond is $R_{L,t}/(R_{L,t} - \kappa)$, which we will match to the average duration of 10-year Treasury Bills. Notice also that the price of a bond issued s periods before is given by $P_{L,t}(s) = \kappa^s P_{L,t}$, which will be used to write the flow budget constraint as a function of the stock of total long-term debt, B_t^L, instead of the current period's purchases of long-term debt. As in standard models, short-term assets B_t are one-period bonds, purchased at time t, which pay a nominal return R_t at time $t+1$.

Households are ordered on a continuum of measure 1. A fraction ω_u of households (unrestricted, or u) trades in both short-term (one-period) and long-term (L-period) bonds. The remaining fraction $\omega_r = 1 - \omega_u$ (restricted, or r) only trades in long-term bonds. Additionally, unrestricted households pay a transaction cost ζ_t per-unit of long-term bond purchased while restricted households do not.

The flow budget constraint differs depending on whether the household is unrestricted or restricted. For an unrestricted household who can trade both short and long-term bonds, we have

$$P_t C_t^u + B_t^u + (1+\zeta_t) P_{L,t} B_t^{L,u} \leq R_{t-1} B_{t-1}^u + \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{L,u} + W_t^u(i) L_t^u(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^u. \tag{A.21}$$

where $\zeta_t P_{L,t} B_t^{L,u}$ is paid to the financial institution who redistributes the proceeds $\mathcal{P}_t^{fi}$ to the household. For a restricted household who can only trade in long-term securities but does not pay transaction costs, we have

$$P_t C_t^r + P_{L,t} B_t^{L,r} \le \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{L,r} + W_t^r(i) L_t^r(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^r. \tag{A.22}$$

In (A.21) and (A.22), P_t is the price of the final consumption good, $W_t^j(i)$ is the wage set by a household of type $j = \{u,r\}$ who supplies labour of type i, $\mathcal{P}_t$ and $\mathcal{P}_t^{cp}$ are profits from ownership of intermediate goods producers and capital producers, respectively, and T_t^j are lump-sum taxes.

One advantage of assuming that the entire stock of long-term government bonds consists of perpetuities of this type is that the price in period t of a bond issued s periods ago $P_{L,t}(s)$ is a function of the coupon and the current price

$$P_{L,t}(s) = \kappa^s P_{L,t}.$$

This relation allows us to rewrite the household budget constraint in a more convenient recursive formulation. One bond of this type that has been issued $s-1$ periods ago is equivalent to κ^{s-1} new bonds. By no arbitrage at time $t-1$

$$P_{L,t-1} B_{t-1}^L = \sum_{s=1}^{\infty} P_{L,t}(s) B_{t-s}^L$$

$$P_{L,t-1} B_{t-1}^L = \sum_{s=1}^{\infty} P_{L,t-1} \kappa^{s-1} B_{t-s}^L$$

$$B_{t-1}^L = \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^L$$

at time t, B_{t-1}^L is worth $B_{t-1}^L(1 + \kappa P_{L,t}) = B_{t-1}^L\{1 + [\kappa/(R_{L,t} - \kappa)]\} = P_{L,t} R_{L,t} B_{t-1}^L$.

The budget constraint of an unrestricted household becomes

$$P_t C_t^u + B_t^u + (1+\zeta_t) P_{L,t} B_t^{L,u} \le R_{t-1} B_{t-1}^u + P_{L,t} R_{L,t} B_{t-1}^{L,u} + W_t^u(i) L_t^u(i) + \mathcal{P}_t^u + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^u. \tag{A.23}$$

For a restricted household, we have

$$P_t C_t^r + P_{L,t} B_t^{L,r} \le P_{L,t} R_{L,t} B_{t-1}^{L,r} + W_t^r(i) L_t^r(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^r, \tag{A.24}$$

where $R_{L,t}$ is the gross yield to maturity at time t on the long-term bond[1]

$$R_{L,t} = \frac{1}{P_{L,t}} + \kappa.$$

Household j consumption-saving decisions are then the result of the maximisation of (A.25) subject to (A.23) if $j = u$ or (A.24) if $j = r$.

Households enjoy consumption $C_{j,t}$ and dislike hours worked $L_{j,t}$. The objective function for all households is

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta_j^s b_{j,t+s} \left[\frac{\left(\dfrac{C_{t+s}^j}{Z_{t+s}} - h \dfrac{C_{t+s-1}^j}{Z_{t+s-1}} \right)^{1-\sigma_j}}{1-\sigma_j} - \frac{\varphi_{t+s}^j L_{t+s}^j(i)^{1+\nu}}{1+\nu} \right], \tag{A.25}$$

where $j = \{u,r\}$, $\beta_j \in (0,1)$ is the individual discount factor (which may differ between restricted

[1] We match the duration of this bond $R_{L,t}/(R_{L,t} - \kappa)$ to the average duration of 10-year US Treasury Bills.

and unrestricted households), $\sigma_j > 0$ is the individual coefficient of relative risk aversion (which may also differ between the different types of households), $\nu \geq 0$ is the inverse elasticity of labour supply, b_t^j is a preference shock to individual j and ψ_t is a labour supply shock.

Define $\Xi_t^{p,u}$ as the Lagrange multiplier associated with the budget constraint (A.23) and $\Xi_t^{p,r}$ the Lagrange multiplier associated with the budget constraint (A.24). Households perfectly share their consumption within their groups (restricted and unrestricted). This assumption implies that the multipliers $\Xi_t^{p,u}$ and $\Xi_t^{p,r}$ are the same for all households of a certain type in all periods and across all states of nature.

The first-order conditions for consumption and bond holdings for an unrestricted household are

$$\begin{aligned}[\partial C_t^u] :& \frac{1}{P_t}\left\{\frac{b_t^u}{Z_t}\left(\frac{C_t^u}{Z_t} - h\frac{C_{t-1}^u}{Z_{t-1}}\right)^{-\sigma_u}\right. \\ &\left. -\beta_u h \mathbb{E}_t\left[\frac{b_{t+1}^u}{Z_t}\left(\frac{C_{t+1}^u}{Z_{t+1}} - h\frac{C_t^u}{Z_t}\right)^{-\sigma_u}\right]\right\} = \Xi_t^{p,u},\end{aligned} \tag{A.26}$$

$$[\partial B_t(u)] : \Xi_t^{p,u} = \beta_u R_t \mathbb{E}_t(\Xi_{t+1}^{p,u}), \tag{A.27}$$

$$[\partial B_t^L(u)] : \frac{1+\zeta_t}{R_{L,t} - \kappa}\Xi_t^{p,u} = \beta_u \mathbb{E}_t\left(\frac{R_{L,t+1}}{R_{L,t+1} - \kappa}\Xi_{t+1}^{p,u}\right). \tag{A.28}$$

The first-order conditions for consumption and bond holdings for a restricted household are

$$\begin{aligned}[\partial C_t(r)] :& \frac{1}{P_t}\left\{\frac{b_t^r}{Z_t}\left(\frac{C_t^r}{Z_t} - h\frac{C_{t-1}^r}{Z_{t-1}}\right)^{-\sigma_r}\right. \\ &\left. -\beta_r h \mathbb{E}_t\left[\frac{b_{t+1}^r}{Z_t}\left(\frac{C_{t+1}^r}{Z_{t+1}} - h\frac{C_t^r}{Z_t}\right)^{-\sigma_r}\right]\right\} = \Xi_t^{p,r},\end{aligned} \tag{A.29}$$

$$[\partial B_t^L(r)] : \frac{1}{R_{L,t} - \kappa}\Xi_t^{p,r} = \beta_r \mathbb{E}_t\left(\frac{R_{L,t+1}}{R_{L,t+1} - \kappa}\Xi_{t+1}^{p,r}\right). \tag{A.30}$$

Households are monopolistic suppliers of labour inputs $L_t(i)$, which perfectly competitive labour agencies aggregate into a homogeneous labour composite L_t according to the technology

$$L_t = \left[\int_0^1 L_t(i)^{1/1+\lambda_w}\mathrm{d}i\right]^{1+\lambda_w}, \tag{A.31}$$

where $\lambda_w \geq 0$ is the steady-state wage markup. The first-order condition for the demand of labour input i is

$$L_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-(1+\lambda_w)/\lambda_w} L_t. \tag{A.32}$$

Combining this condition with the zero-profit condition for labour agencies, we obtain an expression for the aggregate wage index W_t as a function of the wage specific to the ith labour input

$$W_t = \left[\int_0^1 W_t(i)^{-\frac{1}{\lambda_w}}\mathrm{d}i\right]^{-\lambda_w}. \tag{A.33}$$

Household members set wages on a staggered basis (Calvo, 1983) subject to the demand for their specific labour input (A.32). The wage gets reset with probability $1 - \zeta_w$ in each period, while with the complementary probability the wage grows at the steady-state rate of inflation and productivity. Formally, the problem for a household member i of type j who can reset his or her wage at time t is

$$\min_{\tilde{W}_t^j(i)} \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \beta_j)^s b_{t+s}^j \frac{\varphi_{t+s}^j}{1+\nu_j} L_{t+s}^j(i)^{1+\nu_j}, \tag{A.34}$$

subject to the budget constraint (A.23) or (A.24), the demand for labour (A.32) and the wage updating scheme

$$W_{t+s}^j(i) = (\Pi e^{\gamma})^s \tilde{W}_t^j(i). \tag{A.35}$$

The first-order condition for this problem is

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \beta_j)^s \Xi_{t+s}^{p,j} L_{t+s}^j(i) \left[(\Pi e^{\gamma})^s \tilde{W}_t^j(i) - (1+\lambda_w) \frac{b_{t+s}^j \varphi_{t+s}^j L_{t+s}^j(i)^{\nu_j}}{\Xi_{t+s}^{p,j}} \right] = 0. \tag{A.36}$$

In the absence of nominal rigidities, this condition would amount to setting the real wage as a markup over the marginal rate of substitution between consumption and leisure.

All agents of type $j = u,r$ resetting their wage face an identical problem. We focus on the symmetric equilibrium in which all agents of type j that can readjust their wage choose the same $\tilde{W}_t^j$, in which case we get

$$(\tilde{W}_t^j)^{1+\frac{1+\lambda_w}{\lambda_w}\nu_j} = (1+\lambda_w) \frac{\mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \beta_j)^s b_{t+s}^j \varphi_{t+s}^j (\Pi e^{\gamma})^{-s\frac{1+\lambda_w}{\lambda_w}(1+\nu_j)} W_{t+s}^{\frac{1+\lambda_w}{\lambda_w}(1+\nu_j)} L_{t+s}^{1+\nu_j}}{\mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \beta_j)^s \Xi_{t+s}^{p,j} (\Pi e^{\gamma})^{s\left(1-\frac{1+\lambda_w}{\lambda_w}\right)} W_{t+s}^{\frac{1+\lambda_w}{\lambda_w}} L_{t+s}}, \tag{A.37}$$

for $j = u,r$.

Therefore, the aggregate wage index (A.33) can be written as

$$W_t = \left\{ (1-\zeta_w) \left[\omega_u \left(\tilde{W}_t^u\right)^{-\frac{1}{\lambda_w}} + \omega_r \left(\tilde{W}_t^r\right)^{-\frac{1}{\lambda_w}} \right] + \zeta_w (\Pi e^{\gamma} W_{t-1})^{-\frac{1}{\lambda_w}} \right\}^{-\lambda_w}. \tag{A.38}$$

A.5. *Government Policies*

The central bank follows a conventional feedback interest rate rule (Taylor, 1993) with smoothing

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_m} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t/Y_{t-4}}{e^{4\gamma}}\right)^{\phi_y} \right]^{1-\rho_m} \mathrm{e}^{\epsilon_{m,t}},$$

where $\rho_m \in (0,1)$, $\phi_\pi > 1$ and $\phi_y \geq 0$.

The presence of long-term bonds modifies the standard government budget constraint

$$B_t + P_{L,t} B_t^L = R_{t-1,t} B_{t-1} + (1 + \kappa P_{L,t}) B_{t-1}^L + P_t G_t - T_t. \tag{A.39}$$

The left-hand side of expression (A.39) is the market value, in nominal terms, of the total amount of bonds (short-term and long-term) issued by the government at time t. The right-hand side features the cost of servicing bonds maturing at time t as well as spending G_t and taxes T_t.

We assume that the government controls the supply of long-term bond following a simple autoregressive rule

$$\frac{P_{L,t} B_t^L}{P_t Z_t} = S \left(\frac{P_{L,t-1} B_{t-1}^L}{P_{t-1} Z_{t-1}} \right)^{\rho_B} \mathrm{e}^{\epsilon_{B,t}}, \tag{A.40}$$

where $\rho_B \in (0,1)$ and $\epsilon_{B,t}$ is an i.i.d. exogenous shock. S is whatever constant needed to make the above equation an identity at the steady state. We interpret LSAP programmes as shocks to the outstanding government long-term liabilities compared with the historical behaviour of these series.

Finally, we set taxes according to the feedback rule

$$\frac{T_t}{P_t Z_t} - \frac{G_t}{Z_t} \equiv \Phi_{z,t} = \Phi \left(\frac{\frac{1}{R_{L,t-1} - \kappa} B^L_{Z,t-1} + B_{Z,t-1}}{\frac{1}{R_L - \kappa} B^L_Z + B_Z} \right)^{\phi_T} e^{\epsilon_{T,t}}, \tag{A.41}$$

where $\epsilon_{T,t}$ follows a stationary AR(1) process and the term in parenthesis on the right-hand side is the ratio of total debt value in period t to its steady-state value.

A.6. *Term Premium and Preferred Habitat*

Our baseline formulation of the relation between transaction costs and the quantity of debt is

$$\zeta_t = \left(\frac{P_{L,t} B^L_t}{P_t Z_t} \right)^{\rho_\zeta} \exp(\epsilon_{\zeta,t}).$$

The Euler Equation of an *unrestricted* household for investing in long-term bonds is

$$(1 + \zeta_t) P_{L,t} \Xi^{p,u}_t = \beta_u \mathbb{E}_t (P_{L,t+1} R_{L,t+1} \Xi^{p,u}_{t+1}). \tag{A.42}$$

Define $P^{EH}_{L,t}$ and $R^{EH}_{L,t}$ the price and yield to maturity of the long-term bond that would arise in the absence of transaction costs, holding constant the path for the marginal utility of consumption. In defining $R^{EH}_{L,t}$, we also adjust the parameter κ so that in steady state the counterfactual long-term bond has the same maturity of the bond in the model with transaction costs, that is

$$D_L = \frac{R_L}{R_L - \kappa} = \frac{R^{EH}_L}{R^{EH}_L - \kappa^{EH}} = D^{EH}_L. \tag{A.43}$$

The counterpart of (A.42) in this counterfactual world is

$$P^{EH}_{L,t} \Xi^{p,u}_t = \beta_u \mathbb{E}_t (P^{EH}_{L,t+1} R^{EH}_{L,t+1} \Xi^{p,u}_{t+1}). \tag{A.44}$$

No arbitrage implies that the counterfactual long-term bond should have the same risk-adjusted return as the long-term bond in the actual economy with transaction costs. Rearranging (A.42) and (A.44) and taking the difference yields

$$\mathbb{E}_t \left\{ \frac{\Xi^{p,u}_{t+1}}{\Xi^{p,u}_t} \left[\frac{P_{L,t+1}}{(1 + \zeta_t) P_{L,t}} R_{L,t+1} - \frac{P^{EH}_{L,t+1}}{P^{EH}_{L,t}} R^{EH}_{L,t+1} \right] \right\} = 0.$$

Up to a first-order approximation, the previous equation becomes

$$\mathbb{E}_t [\hat{P}_{L,t+1} - \hat{P}_{L,t} - \zeta_t + \hat{R}_{L,t+1} - (\hat{P}^{EH}_{L,t+1} - \hat{P}^{EH}_{L,t} + \hat{R}^{EH}_{L,t+1})] = 0.$$

Also up to the first order, from (A.43) the relation between price and yields is

$$\hat{P}_{L,t} = -D_L \hat{R}_{L,t}.$$

We define the risk premium as the difference, in log-deviations from steady state, of the yield to maturity with and without transaction costs

$$\widehat{RP}_t \equiv \hat{R}_{L,t} - \hat{R}^{EH}_{L,t}.$$

We can then combine the approximation of the no arbitrage condition and the relation between price and yield to obtain a first-order forward looking difference equation in the risk premium

$$(D_L - 1)\mathbb{E}_t\widehat{RP}_{t+1} - D_L\widehat{RP}_t + \zeta_t = 0.$$

Because $D_L > 1$, the previous equation can be solved forward to obtain

$$\widehat{RP}_t = \frac{1}{D_L}\sum_{s=0}^{\infty}\left(\frac{D_L - 1}{D_L}\right)^s \mathbb{E}_t\zeta_{t+s},$$

which corresponds to the equation in the text.

A.7. *Aggregation*

A.7.1. *Resource Constraints*

Budget constraint for the unconstrained household:

$$P_tC_t^u + B_t^u + \frac{1+\zeta_t}{R_{L,t} - \kappa}B_t^{L,u} = R_{t-1}B_{t-1}^u + \frac{R_{L,t}}{R_{L,t} - \kappa}B_{t-1}^{L,u} + \int W_t^u(i)L_t^u(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^u.$$

Budget constraint for the constrained household:

$$P_tC_t^r + \frac{1}{R_{L,t} - \kappa}B_t^{L,r} = \frac{R_{L,t}}{R_{L,t} - \kappa}B_{t-1}^{L,r} + \int W_t^r(i)L_t^r(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^r.$$

Government's budget constraint

$$B_t + \frac{1}{R_{L,t} - \kappa}B_t^L = R_{t-1}B_{t-1} + \frac{R_{L,t}}{R_{L,t} - \kappa}B_{t-1}^L + P_tG_t - T_t.$$

Next, realise that

$$\mathcal{P} = \int_t \mathcal{P}(i)\mathrm{d}i = \int P_t(i)Y_t(i)\mathrm{d}i - W_tL_t - R_t^KK_t,$$

where $L_t = \int L_t(i)\mathrm{d}i$ is total labour supplied by the labour packers and demanded by the firms. $K_t = \int K_t(i)\mathrm{d}i$. We substitute the definition of Π_t into household's budget constraints and realise that the profit of labour packer and good packer is zero.

It must be the case that

$$W_tL_t = \int W_t^u(i)L_t^u(i)\mathrm{d}i + \int W_t^r(i)L_t^r(i)\mathrm{d}i$$

and

$$P_tY_t = \int P_t(i)Y_t(i)\mathrm{d}i.$$

The capital producer's profit is

$$R_t^KK_t - P_ta(u_t)\bar{K}_{t-1} - P_tI_t.$$

The financial institution's profit is

$$\mathcal{P}_t^{fi} = \varpi_u\frac{\zeta_t}{R_{L,t} - \kappa}B_t^{L,u}.$$

Finally the budget constraint is

$$\varpi_uC_t^u + \varpi_rC_t^r + G_t + a(u_t)\bar{K}_{t-1} + I_t = Y_t. \tag{A.45}$$

A.7.2. *Exogenous Processes*

The model is supposed to be fitted to data on output, consumption, investment, employment, wages, nominal interest rates and market value of bonds.

- Technology process: let $z_t = \ln(e^{-\gamma} Z_t / Z_{t-1})$

$$z_t = \rho_z z_t + \epsilon_{z,t}. \tag{A.46}$$

- Preference for leisure:

$$\ln\varphi_t = \rho_\varphi \ln\varphi_{t-1} + \epsilon_{\varphi,t}. \tag{A.47}$$

- Price Mark-up shock:

$$\ln\lambda_{f,t} = \epsilon_{\lambda,t}. \tag{A.48}$$

- Capital adjustment cost process:

$$\ln\mu_t = \rho_\mu \ln \mu_{t-1} + \epsilon_{\mu,t}. \tag{A.49}$$

- Intertemporal preference shifter:

$$\ln b_t = \rho_b \ln b_{t-1} + \epsilon_{b,t}. \tag{A.50}$$

- Government spending process:

$$\ln g_t = \rho_g \ln g_{t-1} + \epsilon_{g,t}. \tag{A.51}$$

- Monetary Policy Shock $\epsilon_{m,t}$.
- Exogenous risk premium shock:

$$\epsilon_{\zeta,t} = \rho_\zeta \epsilon_{\zeta,t-1} + \epsilon_{\zeta,t}. \tag{A.52}$$

- Fiscal shock $\epsilon_{T,t}$
- Long-term bond supply shock $\epsilon_{B,t}$

Appendix B. Normalised Equations

Consider the following normalisations:

- $r_t^k \equiv R_t^k / P_t$; $w_{z,t} \equiv W_t/(Z_t P_t)$; $mc_t \equiv MC_t/P_t$; $q_t \equiv Q_t/P_t$
- $\Xi_t^j \equiv \Xi_t^p(j) Z_t P_t$, $\forall j$
- $x_{z,t} \equiv x_t/Z_t$, $\forall x_t$, except for the cases below
- $B_{z,t} \equiv B_t/(P_t Z_t)$; $B_{z,t}^L \equiv B_t^L/(P_t Z_t)$; $G_{z,t} \equiv G_t/Z_t$; $T_{z.t} \equiv T_t/(P_t Z_t)$

Real marginal cost

$$mc_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}(r_t^k)^\alpha w_{z,t}^{1-\alpha}. \tag{B.1}$$

Capital demand

$$K_{z,t} = \frac{\alpha}{1-\alpha} \frac{w_{z,t}}{r_t^k} L_t. \tag{B.2}$$

Technology

$$Y_{z,t} = K_{z,t}^{\alpha} L_t^{1-\alpha}. \tag{B.3}$$

Price setting

$$\tilde{p}_t = \frac{\omega_u X_t^{n,u} + \omega_r X_t^{n,r}}{\omega_u X_t^{d,u} + \omega_r X_t^{d,r}}. \tag{B.4}$$

with

$$X_t^{pn,j} = \Xi_t^j Y_{z,t}(1+\lambda_f)\lambda_{f,t} mc_t + \beta_j \zeta_p \mathbb{E}_t \left[\left(\frac{\Pi_{t+1}}{\Pi} \right)^{\frac{1+\lambda_f}{\lambda_f}} X_{t+1}^{pn,j} \right], j = u, r, \tag{B.5}$$

$$X_t^{pd,j} = \Xi_t^j Y_{z,t} + \beta_j \zeta_p \mathbb{E}_t \left[\left(\frac{\Pi_{t+1}}{\Pi} \right)^{\frac{1}{\lambda_f}} X_{t+1}^{pd,j} \right], j = u, r. \tag{B.6}$$

LOM prices

$$1 = (1-\zeta_p) \left(\frac{\omega_u X_t^{pn,u} + \omega_r X_t^{pn,r}}{\omega_u X_t^{pd,u} + \omega_r X_t^{pd,r}} \right)^{-\frac{1}{\lambda_f}} + \zeta_p \left(\frac{\Pi}{\Pi_t} \right)^{-\frac{1}{\lambda_f}}. \tag{B.7}$$

Effective capital

$$K_{z,t} = e^{-\gamma - z_t} u_t \bar{K}_{z,t-1}. \tag{B.8}$$

Law of motion of capital

$$\bar{K}_{z,t} = (1-\delta) e^{-\gamma - z_t} \bar{K}_{z,t-1} + \mu_t \left[1 - S\left(\mathrm{e}^{\gamma + z_t} \frac{I_{z,t}}{I_{z,t-1}} \right) \right] I_{z,t}. \tag{B.9}$$

Capital utilisation

$$r_t^k = a'(u_t). \tag{B.10}$$

Law of motion of Q

$$q_t = \mathbb{E}_t \left\{ \frac{\omega_u \beta_u \Xi_{t+1}^u + \omega_r \beta_r \Xi_{t+1}^r}{\omega_u \Xi_t^u + \omega_r \Xi_t^r} e^{-\gamma - z_{t+1}} [r_{t+1}^k u_{t+1} - a(u_{t+1}) + (1-\delta) q_{t+1}] \right\}. \tag{B.11}$$

Investment decision

$$\begin{aligned} 0 = {} & -1 + q_t \mu_t \left[1 - S\left(e^{\gamma + z_t} \frac{I_{z,t}}{I_{z,t-1}} \right) \right] - q_t \mu_t S'\left(e^{\gamma + z_t} \frac{I_{z,t}}{I_{z,t-1}} \right) \mathrm{e}^{\gamma + z_t} \frac{I_{z,t}}{I_{z,t-1}} \\ & + \mathbb{E}_t \left[\frac{\omega_u \beta_u \Xi_{t+1}^u + \omega_r \beta_r \Xi_{t+1}^r}{\omega_u \Xi_t^u + \omega_r \Xi_t^r} \mathrm{e}^{-\gamma - z_{t+1}} q_{t+1} \mu_{t+1} S'\left(\mathrm{e}^{\gamma + z_{t+1}} \frac{I_{z,t+1}}{I_{z,t}} \right) \left(e^{\gamma + z_{t+1}} \frac{I_{z,t+1}}{I_{z,t}} \right)^2 \right]. \end{aligned} \tag{B.12}$$

Marginal Utilities for each type:

$$\Xi_t^j = b_t^j (C_{z,t}^j - h C_{z,t-1}^j)^{-\sigma_j} - \beta_j h \mathbb{E}_t \left[b_{t+1}^j \left(C_{z,t+1}^j - h C_{z,t}^j \right)^{-\sigma_j} \right], j = u, r. \tag{B.13}$$

Euler equation: Unconstrained, short

$$\Xi_t^u = \beta_u R_t \mathbb{E}_t [e^{-\gamma - z_{t+1}} \Xi_{t+1}^u \Pi_{t+1}^{-1}]. \tag{B.14}$$

Euler equation: Unconstrained, long

$$(1+\zeta_t)\Xi_t^u = \beta_u \mathbb{E}_t\left[\Xi_{t+1}^u e^{-\gamma - z_{t+1}} \Pi_{t+1}^{-1} \frac{R_{L,t}-\kappa}{R_{L,t+1}-\kappa} R_{L,t+1}\right]. \tag{B.15}$$

Euler equation: Constrained, long

$$\Xi_t^r = \beta_r \mathbb{E}_t\left[\Xi_{t+1}^r e^{-\gamma - z_{t+1}} \Pi_{t+1}^{-1} \frac{R_{L,t}-\kappa}{R_{L,t+1}-\kappa} R_{L,t+1}\right]. \tag{B.16}$$

Wage setting

$$(\tilde{w}_{z,t}^j)^{1+\frac{1+\lambda_w}{\lambda_w}v_j} = \frac{X_t^{wn,j}}{X_t^{wd,j}}, j = u, r. \tag{B.17}$$

$$X_t^{wn,j} = (1+\lambda_w) b_t^j \varphi_t^j L_t^{1+v_j} w_{z,t}^{\frac{1+\lambda_w}{\lambda_w}(1+v_j)} + \zeta_w \beta_j \mathbb{E}_t\left[\left(\frac{\Pi_{t+1}\mathrm{e}^{z_{t+1}}}{\Pi}\right)^{\frac{1+\lambda_w}{\lambda_w}(1+v_j)} X_{t+1}^{wn,j}\right], j = u, r, \tag{B.18}$$

$$X_t^{wd,j} = \Xi_t^j L_t w_{z,t}^{\frac{1+\lambda_w}{\lambda_w}} + \zeta_w \beta_j \mathbb{E}_t\left[\left(\frac{\Pi_{t+1}\mathrm{e}^{z_{t+1}}}{\Pi}\right)^{\frac{1}{\lambda_w}} X_{t+1}^{wd,j}\right], j = u, r. \tag{B.19}$$

Law of motion of real wages

$$w_{z,t} = \left[(1-\zeta_w)\left(\omega_u\left(\frac{X_t^{wn,u}}{X_t^{wd,u}}\right)^{-\frac{1}{\lambda_w}\frac{1}{1+\frac{1+\lambda_w}{\lambda_w}v_u}} + \omega_r\left(\frac{X_t^{wn,r}}{X_t^{wd,r}}\right)^{-\frac{1}{\lambda_w}\frac{1}{1+\frac{1+\lambda_w}{\lambda_w}v_r}}\right) + \zeta_w\left(\frac{\Pi w_{z,t-1}}{\Pi_t e^{z_t}}\right)^{-\frac{1}{\lambda_w}}\right]^{-\lambda_w}. \tag{B.20}$$

Budget constraint

$$B_{z,t} + \frac{1}{R_{L,t}-\kappa} B_{z,t}^L = \frac{R_{t-1}}{e^{\gamma+z_t}\Pi_t} B_{z,t-1} + \frac{R_{L,t}}{R_{L,t}-\kappa}\frac{1}{\mathrm{e}^{\gamma+z_t}\Pi_t} B_{z,t-1}^L + G_{z,t} - T_{z,t}. \tag{B.21}$$

Long-term bond policy

$$P_{L,t}B_{z,t}^L = S(P_{L,t-1}B_{z,t-1}^L)^{\rho_B}\mathrm{e}^{\epsilon_{B,t}}. \tag{B.22}$$

Transfers feedback rule

$$T_{z,t} - G_{z,t} \equiv \Phi_{z,t} = \Phi\left(\frac{\frac{1}{R_{L,t-1}-\kappa}B_{Z,t-1}^L + B_{Z,t-1}}{\frac{1}{R_L-\kappa}B_Z^L + B_Z}\right)^{\phi_T} \exp^{\epsilon_{T,t}}. \tag{B.23}$$

Monetary policy

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_m}\left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\left(\frac{Y_{z,t}}{Y_{z,t-4}}e^{z_{t-3}+\cdots+z_t}\right)^{\phi_y}\right]^{1-\rho_m} e^{\epsilon_{m,t}}. \tag{B.24}$$

Term premium

$$\zeta_t \equiv \zeta(P_{L,t}B_{z,t}^L, \epsilon_{\zeta,t}). \tag{B.25}$$

Aggregate resources constraint

$$\omega_u C_{z,t}^u + \omega_r C_{z,t}^r + I_{z,t} + G_{z,t} + e^{-\gamma-z_t}a(u_t)\bar{K}_{z,t-1} = Y_{z,t}. \tag{B.26}$$

Appendix C. Model Steady State

In steady state, the log of productivity grows at the constant rate γ and inflation is constant and equal to Π.

We choose a functional form for $a(u_t)$ such that $u = 1$ in steady state and $a(1) = 0$ (See Christiano *et al.*, 2011). Furthermore, we consider:

$$Y_z = 1,$$

$$v_u = v_r = v,$$

$$S(e^{\gamma}) = S'(e^{\gamma}) = 0,$$

and estimate

$$\frac{C^u}{C^r}, \frac{\Xi^u}{\Xi^r}$$

and let the levels of b^u and b^r be whatever they need to be to allow these ratios to be consistent with each other and the resources constraint in levels.

Euler equations imply

$$1 = \beta_u R e^{-\gamma} \Pi^{-1}, \tag{C.1}$$

$$(1 + \zeta) = \frac{R_L}{R}, \tag{C.2}$$

$$\beta_u = \beta_r (1 + \zeta). \tag{C.3}$$

Risk premium relation determines level of long debt

$$B_z^{LMV} = \zeta^{-1}(\zeta). \tag{C.4}$$

Govt BC determines taxes

$$T_z = G_z - \left(1 - \beta_u^{-1}\right) B_z - \left(\frac{1}{R_L^L - \kappa} - \frac{R_L^L}{R_L^L - \kappa} \frac{1}{e^{\gamma} \Pi} \right) B_z^L. \tag{C.5}$$

Unit MEI shock implies

$$1 = q. \tag{C.6}$$

Unit utilisation implies

$$r^k = a'(1), \tag{C.7}$$

which pins down $a'(1)$ given r^k.

FOC for investment implies

$$r^k = \bar{\beta}^{-1} e^{\gamma} - (1 - \delta), \tag{C.8}$$

with

$$\bar{\beta} \equiv \frac{\omega_u \beta_u \Xi^u + \omega_r \beta_r \Xi^r}{\omega_u \Xi^u + \omega_r \Xi^r} = \frac{\omega_u \beta_u \frac{\Xi^u}{\Xi^r} + \omega_r \beta_r}{\omega_u \frac{\Xi^u}{\Xi^r} + \omega_r}$$

which is a function of Ξ^u/Ξ^r. Hence, r^k is also known given the estimate/calibration of Ξ^u/Ξ^r.

Price setting implies

$$mc = \frac{1}{1+\lambda_f}. \tag{C.9}$$

Definition of marginal cost implies

$$w_z = \tilde{w}_z (r^k)^{-\frac{\alpha}{1-\alpha}}, \tag{C.10}$$

with

$$\tilde{w}_z \equiv (1+\lambda_f)^{-\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha).$$

Technology function implies

$$L = K_z^{-\frac{\alpha}{1-\alpha}}$$

and plug into capital demand implies

$$K_z = \tilde{K}_z (r^k)^{-1}, \tag{C.11}$$

with

$$\tilde{K}_z = \frac{\alpha}{1+\lambda_f}$$

which then implies that

$$L = \tilde{L}(r^k)^{\frac{\alpha}{1-\alpha}}, \tag{C.12}$$

with

$$\tilde{L} \equiv \left(\frac{\alpha}{1+\lambda_f}\right)^{-\frac{\alpha}{1-\alpha}}.$$

Effective capital

$$\bar{K}_z = e^{\gamma} \tilde{K}_z (r^k)^{-1}. \tag{C.13}$$

Investment

$$I_z = [e^{\gamma} - (1-\delta)] \tilde{K}_z (r^k)^{-1}. \tag{C.14}$$

Resources constraint

$$\omega_u C_z^u + \omega_r C_z^r = 1 - I_z - G_z, \tag{C.15}$$

and given the ratio of consumptions, we get

$$C_z^r = \frac{1 - I_z - G_z}{\omega_u \frac{C_z^u}{C_z^r} + \omega_r}, \tag{C.16}$$

$$C_z^u = \frac{C_z^u}{C_z^r} C_z^r. \tag{C.17}$$

Furthermore, notice that

$$\frac{X^{pn,u}}{X^{pn,r}} = \frac{X^{pd,u}}{X^{pd,r}} = \frac{\Xi^u}{\Xi^r}\frac{1-\beta_r\zeta_p}{1-\beta_u\zeta_p}, \tag{C.18}$$

which is known.

For the wages, we have

$$\frac{X^{wn,j}}{X^{wd,j}} = (1+\lambda_w)L^{\nu} w_z^{\frac{1+\lambda_w}{\lambda_w}\nu}\frac{b^j\varphi^j}{\Xi^j}$$

and in the log-linearisation, we will need the ratio

$$\frac{\frac{X^{wn,u}}{X^{wd,u}}}{\frac{X^{wn,r}}{X^{wd,r}}} = \frac{b^u\varphi^u}{b^r\varphi^r}\frac{\Xi^r}{\Xi^u}, \tag{C.19}$$

or

$$\chi_{wu} = \frac{\omega_u}{\omega_u + \omega_r\left(\frac{b^u\varphi^u}{b^r\varphi^r}\frac{\Xi^r}{\Xi^u}\right)^{\frac{1}{\lambda_w}\frac{1}{1+[(1+\lambda_w)/\lambda_w]\nu}}}$$

which, given b^u/b^r and Ξ^u/Ξ^r is given by φ^u/φ^r. Let us then estimate/calibrate this ratio, χ_{wu}, which has to be between 0 and 1.

Rest of steady-state relations (not explicitly needed for the numerical analysis)

$$X^{pn,j} = \frac{\Xi^j Y_z(1+\lambda_f)mc}{1-\beta_j\zeta_p}, j = u, r, \tag{C.20}$$

$$X^{pd,j} = \frac{\Xi^j Y_z}{1-\beta_j\zeta_p}, j = u, r, \tag{C.21}$$

$$\Xi^j = b^j(1-\beta_j h)(1-h)^{-\sigma_j}(C_z^j)^{-\sigma_j}, j = u, r, \tag{C.22}$$

$$X^{wn,j} = (1+\lambda_w)\frac{b^j\varphi^j L^{1+\nu_j} w_z^{\frac{1+\lambda_w}{\lambda_w}(1+\nu)}}{1-\zeta_w\beta_j}, j = u, r, \tag{C.23}$$

$$X^{wd,j} = \frac{\Xi^j L w_z^{\frac{1+\lambda_w}{\lambda_w}}}{1-\zeta_w\beta_j}, j = u, r, \tag{C.24}$$

$$w_z = \left[\omega_u\left(\frac{X^{wn,u}}{X^{wd,u}}\right)^{-\frac{1}{\lambda_w}\frac{1}{1+\frac{1+\lambda_w}{\lambda_w}\nu}} + \omega_r\left(\frac{X^{wn,r}}{X^{wd,r}}\right)^{-\frac{1}{\lambda_w}\frac{1}{1+\frac{1+\lambda_w}{\lambda_w}\nu}}\right]^{-\lambda_w}. \tag{C.25}$$

Appendix D. Log-linear Approximation

Consider in general that

$$\hat{x}_t \equiv \ln(x_t/x)$$

for any variable x, except for $\hat{\zeta}_t \equiv \ln\left(\frac{1+\zeta_t}{1+\zeta}\right)$, $r_t \equiv \ln(R_t/R)$, and $r_{L,t} \equiv \ln(R_{L,t}/R_L)$

Real marginal cost

$$\widehat{mc}_t = \alpha \hat{r}_t^k + (1-\alpha)\hat{w}_{z,t}. \tag{D.1}$$

Capital demand

$$\hat{K}_{z,t} = \hat{w}_{z,t} - \hat{r}_t^k + \hat{L}_t. \tag{D.2}$$

Technology

$$\hat{Y}_{z,t} = \alpha \hat{K}_{z,t} + (1-\alpha)\hat{L}_t. \tag{D.3}$$

Price setting

$$\hat{X}_t^{pn,j} = (1-\beta_j\zeta_p)\left(\hat{\Xi}_t^j + \hat{Y}_{z,t} + \hat{\lambda}_{f,t} + \widehat{mc}_t\right) + \beta_j\zeta_p \mathrm{E}_t\left(\frac{1+\lambda_f}{\lambda_f}\pi_{t+1} + \hat{X}_{t+1}^{pn,j}\right), j = u, r, \tag{D.4}$$

$$\hat{X}_t^{pd,j} = (1-\beta_j\zeta_p)\left(\hat{\Xi}_t^j + \hat{Y}_{z,t}\right) + \beta_j\zeta_p \mathrm{E}_t\left(\frac{1}{\lambda_f}\pi_{t+1} + \hat{X}_{t+1}^{pd,j}\right), j = u, r. \tag{D.5}$$

LOM prices

$$\pi_t = \frac{1-\zeta_p}{\zeta_p}\left[\chi_{pu}\hat{X}_t^{pn,u} + (1-\chi_{pu})\hat{X}_t^{pn,r} - \chi_{pu}\hat{X}_t^{pd,u} - (1-\chi_{pu})\hat{X}_t^{pd,r}\right], \tag{D.6}$$

with

$$\chi_{pu} \equiv \frac{\omega_u}{\omega_u + \omega_r \frac{1-\beta_u\zeta_p}{1-\beta_r\zeta_p}(\Xi^u/\Xi^r)^{-1}},$$

Effective capital

$$\hat{K}_{z,t} = -z_t + \hat{u}_t + \hat{\bar{K}}_{z,t-1}. \tag{D.7}$$

Law of motion of capital

$$\hat{\bar{K}}_{z,t} = (1-\delta)e^{-\gamma}(\hat{\bar{K}}_{z,t-1} - z_t) + [1-(1-\delta)e^{-\gamma}](\hat{\mu}_t + \hat{I}_{z,t}). \tag{D.8}$$

Capital utilisation

$$\hat{r}_t^k = \frac{a''(1)}{r^k}\hat{u}_t. \tag{D.9}$$

Law of motion of Q

$$\begin{aligned}\hat{q}_t = {} & \bar{\beta}e^{-\gamma}\mathrm{E}_t[r^k\hat{r}_{t+1}^k + (1-\delta)\hat{q}_{t+1}] - \mathrm{E}_t\hat{z}_{t+1} \\ & + \mathrm{E}_t\left[q_u\left(\frac{1+\zeta}{1+q_u\zeta}\hat{\Xi}_{t+1}^u - \hat{\Xi}_t^u\right) + (1-q_u)\left(\frac{1}{1+q_u\zeta}\hat{\Xi}_{t+1}^r - \hat{\Xi}_t^r\right)\right],\end{aligned} \tag{D.10}$$

with

$$q_u \equiv \frac{\omega_u\Xi^u}{\omega_u\Xi^u + \omega_r\Xi^r} = \left(\frac{\bar{\beta}}{\beta_r} - 1\right)\zeta^{-1}.$$

Investment decisions

$$0 = \hat{q}_t + \hat{\mu}_t - e^{2\gamma}S''(\hat{z}_t + \hat{I}_{z,t} - \hat{I}_{z,t-1}) + \bar{\beta}e^{2\gamma}S''\mathbb{E}_t[z_{t+1} + \hat{I}_{z,t+1} - \hat{I}_{z,t}]. \tag{D.11}$$

Marginal Utilities for each type

$$\hat{\Xi}_t^j = \frac{1}{1-\beta_j h}\left[\left(\hat{b}_t^j - \beta_j h \mathrm{E}_t \hat{b}_{t+1}^j\right) - \frac{\sigma_j}{1-h}\left\{(1+\beta_j h^2)\hat{C}_{z,t}^j - \beta_j h \mathrm{E}_t \hat{C}_{z,t+1}^j - h\hat{C}_{z,t-1}^j\right\}\right], j = u, r. \tag{D.12}$$

Euler equation: Unconstrained, short

$$\hat{\Xi}_t^u = r_t + \mathbb{E}_t(\hat{\Xi}_{t+1}^u - z_{t+1} - \pi_{t+1}). \tag{D.13}$$

Euler equation: Unconstrained, long

$$\hat{\zeta}_t + \hat{\Xi}_t^u = \frac{R_L}{R_L - \kappa} r_{L,t} + \mathbb{E}_t\left[\hat{\Xi}_{t+1}^u - z_{t+1} - \pi_{t+1} - \frac{\kappa}{R_L - \kappa} r_{L,t+1}\right]. \tag{D.14}$$

Euler equation: Constrained, long

$$\hat{\Xi}_t^r = \frac{R_L}{R_L - \kappa} r_{L,t} + \mathbb{E}_t\left[\hat{\Xi}_{t+1}^r - z_{t+1} - \pi_{t+1} - \frac{\kappa}{R_L - \kappa} r_{L,t+1}\right]. \tag{D.15}$$

Wage setting

$$\begin{aligned} \hat{X}_t^{wn,j} &= (1-\zeta_w \beta_j)\left[\hat{b}_t^j + \hat{\varphi}_t^j + (1+v)\hat{L}_t + \left(\frac{1+\lambda_w}{\lambda_w}\right)(1+v)\hat{w}_{z,t}\right] \\ &+ \zeta_w \beta_j \mathbb{E}_t\left[\frac{1+\lambda_w}{\lambda_w}(1+v)(\pi_{t+1} + z_{t+1}) + \hat{X}_{t+1}^{wn,j}\right], j = u, r \end{aligned}, \tag{D.16}$$

$$\begin{aligned} \hat{X}_t^{wd,j} &= (1-\zeta_w \beta_j)\left[\hat{\Xi}_t^j + \hat{L}_t + \frac{1+\lambda_w}{\lambda_w}\hat{w}_{z,t}\right] \\ &+ \zeta_w \beta_j \mathbb{E}_t\left[\frac{1}{\lambda_w}(\pi_{t+1} + z_{t+1}) + \hat{X}_{t+1}^{wd,j}\right], j = u, r. \end{aligned} \tag{D.17}$$

Law of motion of real wages

$$\begin{aligned} \hat{w}_{z,t} &= (1-\zeta_w)\frac{1}{1 + \frac{1+\lambda_w}{\lambda_w}v}\left[\chi_{wu}\left(\hat{X}_t^{wn,u} - \hat{X}_t^{wd,u}\right) + (1-\chi_{wu})\left(\hat{X}^{wn,r} - \hat{X}^{wd,r}\right)\right] \\ &+ \zeta_w(\hat{w}_{z,t-1} - \pi_t - z_t), \end{aligned} \tag{D.18}$$

with

$$\chi_{wu} = \frac{\omega_u}{\omega_u + \omega_r w_{ur}^{\frac{1}{\lambda_w + (1+\lambda_w)v}}}$$

Budget constraint

$$\begin{aligned} \hat{B}_{z,t} + \frac{B_z^L/B_z}{R_L - \kappa}\hat{B}_{z,t}^L &= \beta_u^{-1}\left(\hat{B}_{z,t-1} + r_{t-1}\right) + \frac{B_z^L/B_z}{R_L - \kappa}\beta_r^{-1}\hat{B}_{z,t-1}^L \\ &+ \frac{(1 - \mathrm{e}^{-\gamma}\Pi^{-1}\kappa)R_L}{R_L - \kappa}\frac{B_z^L/B_z}{R_L - \kappa} r_{L,t} \\ &+ \frac{G_z}{B_z}\hat{G}_{z,t} - \frac{Y_z}{B_z}\hat{T}_{z,t} - \left(\beta_u^{-1} + \frac{B_z^L/B_z}{R_L - \kappa}\beta_r^{-1}\right)(z_t + \pi_t), \end{aligned} \tag{D.19}$$

with

$$T_{z,t} \equiv T_z + Y_z \hat{T}_{z,t} \Rightarrow \hat{T}_{z,t} = \frac{T_{z,t}}{Y_z} - \frac{T_z}{Y_z}.$$

Long-term bond policy

$$-\frac{R_L}{R_L-\kappa}r_{L,t}+\hat{B}^L_{z,t}=\rho_B(-\frac{R_L}{R_L-\kappa}r_{L,t-1}+\hat{B}^L_{z,t-1})+\epsilon_{B,t}. \tag{D.20}$$

Transfers feedback rule

$$\frac{\hat{T}_{z,t}-G_z\hat{G}_{z,t}}{T_z-G_z}=\phi_T\left[\frac{\hat{B}_{z,t-1}+\frac{1}{R_L-\kappa}(B^L_z/B_z)\hat{B}^L_{z,t-1}-\frac{R_L}{(R_L-\kappa)^2}(B^L_z/B_z)r_{L,t-1}}{1+\frac{1}{R_L-\kappa}(B^L_z/B_z)}\right]+\epsilon_{T,t}. \tag{D.21}$$

Monetary policy

$$r_t=\rho_r r_{t-1}+(1-\rho_r)\left[\phi_\pi\pi_t+\phi_y\left(\hat{Y}_{z,t}-\hat{Y}_{z,t-4}+\sum_{i=0}^{3}z_{t-1}\right)\right]+\epsilon_{m,t}. \tag{D.22}$$

Term premium

$$\hat{\zeta}_t=\zeta'\hat{B}^L_{z,t}+\epsilon_{\zeta,t}. \tag{D.23}$$

Aggregate resources constraint

$$\hat{Y}_{z,t}=\frac{\omega_u C^u_z}{Y_z}\hat{C}^u_{z,t}+\frac{\omega_r C^r_z}{Y_z}\hat{C}^r_{z,t}+\frac{I_z}{Y_z}\hat{I}_{z,t}+\frac{G_z}{Y_z}\hat{G}_{z,t}+\mathrm{e}^{-\gamma}r^k\frac{\bar{K}_z}{Y_z}\hat{u}_t. \tag{D.24}$$

Appendix E. Data

We use quarterly data for the US from the third quarter of 1987 (1987q3) to the third quarter of 2009 (2009q3) for the following seven series: real GDP per capita, hours worked, real wages, core personal consumption expenditures deflator, nominal effective Federal Funds rate, the 10-year Treasury constant maturity yield, and the ratio between long-term and short-term US Treasury debt. All data are extracted from the Federal Reserve Economic Data (FRED) maintained by the Federal Reserve Bank of St. Louis. The mapping of these variables to the states is

$$\begin{aligned}
\Delta Y^{obs}_t &= 100(\gamma+\hat{Y}_{z,t}-\hat{Y}_{z,t-1}+\hat{z}_t),\\
L^{obs}_t &= 100(L+\hat{L}_t),\\
\Delta w^{obs}_t &= 100(\gamma+\hat{w}_{z,t}-\hat{w}_{z,t-1}+\hat{z}_t),\\
\pi^{obs}_t &= 100(\pi+\hat{\pi}_t),\\
r^{obs}_t &= 100(r+\hat{r}_t),\\
r^{obs}_{L,t} &= 100(r_L+\hat{r}_{L,t}),\\
B^{ratio,obs}_t &= \frac{P_L B^L_z}{B_z}(1+\hat{P}_{L,t}+\hat{B}^L_{z,t}-\hat{B}_{z,t}),
\end{aligned}$$

where all state variables are in deviations from their steady-state values, $\pi\equiv\ln(\Pi)$, $r\equiv\ln(R)$ and $r_L\equiv\ln(R_L)$.

We construct real GDP by dividing the nominal GDP series by population and the GDP deflator. The observable ΔY^{obs}_t corresponds to the first difference in logs of this series, multiplied by 100. We measure the labour input by the log of hours of all persons in the non-farm business sector divided by population. Real wages correspond to nominal compensation per hour in the non-farm business sector, divided by the GDP deflator. As for GDP, Δw^{obs}_t is the first difference in logs of this series, multiplied by 100. The quarterly log-difference in the personal consumption expenditures (PCE) core price index is our measure of inflation. We use the effective Federal

Funds Rate as our measure of nominal short-term rate and the 10-year Treasury constant maturity rate as our measure of nominal long-term interest rate. Finally, we identify long-term bonds as US Treasury securities with maturity greater than one year, consistent with the announcement of LSAP II, and construct the ratio to short-term bonds as our measure for the quantity of debt.

Appendix F. Implementing the Commitment to the Zero Lower Bound

In this Section, we describe how we implement the commitment to the zero lower bound. This same approach is also used to guarantee that none of simulation paths violates the non-negative interest rate constraint.

F.1. *Canonical Model*

Consider the economic model in its canonical form, as in Sims (2002):

$$\Gamma_4^{s(t)}(\theta)z_t = \bar{\Gamma}_0^{s(t)} + \Gamma_1^{s(t)}(\theta)z_{t-1} + \Gamma_2^{s(t)}(\theta)\varepsilon_t + \Gamma_3^{s(t)}(\theta)\eta_t, \tag{F.1}$$

where $s(t) \in \{n,zlb\}$ refers to the state of the economy, with n referring to normal times and zlb for times of in which the zero lower bound is binding; z_t is the vector of state variables, whether they are endogenous or exogenous; ε_t is a vector of exogenous i.i.d. innovations; η_t is a vector of endogenous expectational errors; and $\{\Gamma_\iota^{s(t)}(\theta)\}_{\iota=0,1,2,3,4}$ are matrices defining the state space for any given vector of parameters θ.

For simplification of notation, below I will omit the reference to the vector of parameters when writing the matrices.

With some restrictions it is possible to break the system in (F.1) into two blocks: a forward looking one and a backward looking one. So for each equation, we can write:

$$j \in FL : \Gamma_4^{s(t)}(j)E_t z_{t+1} = \Gamma_0^{s(t)}(j) + \Gamma_1^{s(t)}(j)z_t, \tag{F.2}$$

$$i \in BL : \Gamma_4^{s(t)}(i)z_t = \Gamma_0^{s(t)}(i) + \Gamma_1^{s(t)}(i)z_{t-1} + \Gamma_2^{s(t)}(i)\varepsilon_t, \tag{F.3}$$

where i denotes BL equations and j the FL ones.

F.2. *Perfect Foresight Solution Method*

Consider a sequence of periods $\{s(t)\}_{t=0}^{K}$ such that for $t > K$ we have $s(t) = n$ and $\varepsilon_t = 0$ — i.e. n eventually becomes an absorbing state and no additional innovations are expected beyond K. In this case, we can solve for the rational expectations equilibrium (REE) solution backwards.

F.2.1. *Absorbing state*

In normal times, for $t > K$, the REE solution can be represented by

$$z_t = \Phi_0^n + \Phi_1^n z_{t-1} + \Phi_2^n \varepsilon_t. \tag{F.4}$$

F.2.2. *Before the absorbing state*

We need to solve for the REE matrices recursively.

Notice first that for the last period before the absorbing state kicks in, and using (F.4), we can write the forward looking component of the system as

$$\Gamma_4^{s(t)}(j)\mathrm{E}_t(\Phi_0^n + \Phi_1^n z_t + \Phi_2^n \varepsilon_{t+1}) = \Gamma_0^{s(t)}(j) + \Gamma_1^{s(t)}(j)z_t$$

which we can rewrite as

$$[\Gamma_4^{s(t)}(j)\Phi_1^n - \Gamma_1^{s(t)}(j)]z_t = \Gamma_0^{s(t)}(j) - \Gamma_4^{s(t)}(j)(\Phi_0^n + \Phi_2^n\hat{\varepsilon}_{t+1})$$

and combine this with the backward looking to get the full system written as

$$\tilde{\Gamma}_4(t)z_t = \tilde{\Gamma}_0(t) + \tilde{\Gamma}_1(t)z_{t-1} + \tilde{\Gamma}_2(t)\varepsilon_t, \tag{F.5}$$

with

$$\tilde{\Gamma}_4(t) \equiv \begin{bmatrix} \Gamma_4^{s(t)}(j)\Phi_1(t+1) - \Gamma_1^{s(t)}(j) \\ \Gamma_4^{s(t)}(i) \end{bmatrix}, \tag{F.6}$$

$$\tilde{\Gamma}_0(t) \equiv \begin{bmatrix} \Gamma_0^{s(t)}(j) - \Gamma_4^{s(t)}(j)[\Phi_0(t+1) + \Phi_2^n(t+1)\hat{\varepsilon}_{t+1}] \\ \Gamma_0^{s(t)}(i) \end{bmatrix}, \tag{F.7}$$

$$\tilde{\Gamma}_\imath(t) \equiv \begin{bmatrix} 0 \\ \Gamma_\imath^{s(t)}(i) \end{bmatrix}, \text{for } \imath = 1, 2, \tag{F.8}$$

and

$$\Phi_\imath(t+1) = \Phi_\imath^n, \text{for } t = K \text{ and } \imath = 0, 1. \tag{F.9}$$

Now, we can solve this system for z_t and write

$$z_t = \Phi_0(t) + \Phi_1(t)z_{t-1} + \Phi_2(t)\varepsilon_t, \tag{F.10}$$

with

$$\Phi_0(t) \equiv \left[\tilde{\Gamma}_4(t)\right]^{-1}\tilde{\Gamma}_0(t), \tag{F.11}$$

$$\Phi_1(t) \equiv \left[\tilde{\Gamma}_4(t)\right]^{-1}\tilde{\Gamma}_1(t), \tag{F.12}$$

$$\Phi_2(t) \equiv \left[\tilde{\Gamma}_4(t)\right]^{-1}\tilde{\Gamma}_2(t), \tag{F.13}$$

and notice that we need to use a pseudo inverse, to account for the fact that $\tilde{\Gamma}_4(t)$ might not be invertible.

Notice that (F.10) is in the exact same form of (F.4). So, iterating backwards, the system (F.5) and the REE solution (F.10) are valid for $\forall t \leq K$.

F.3. *Implementing the ZLB Commitment*

We use the convention in our simulations that period $t = 0$ is the period in which LSAP is announced and implementation started, and the commitment to the zero lower bound applies to the first four periods, including period $t = 0$. Given the framework just described, then implementing the commitment to the ZLB implies setting a sequence of states $\{s(t)\}_{t=0}^K$ such that $s_t =$ *zlb* for $t =$ 0,1,2,3 and $s_t = n$ for $t > 3$. Then, iterate backwards, starting in period *3* towards the initial period to find the REE solution matrices for periods $t =$ 0,1,2,3. For periods $t > 3$, the solution is the usual one in the absence of policy regime change.

For the *zlb* regime, we have exactly the same equations as in regime *n* but replace the interest rate rule equation with one setting the interest rate to zero.

F.4. *Enforcing Non-Negative Interest Rate*

We can also use this same framework to enforce the non-negative interest rate constraint after the commitment to the zero lower bound is lifted. This is relevant because for some parameter draws we get this constraint to be violated. To accomplish this we use a guess and verify approach.

In the first step, we make the simulation under the assumption that the sequence of states $\{s(t)\}_{t=0}^{K}$ is the one described above. Then, we check for any violations of the non-negative interest rate constraint and switch the regime for those periods from *n* to *zlb*, and solve again for the solution. We keep doing this until there are no violations.

Appendix G. Robustness

This Section considers four robustness exercises. First, we consider the implications of extending the duration of the LSAP programme. Second, we consider a longer commitment to the zero lower bound by the monetary authority. Third, we ask how sensitive the model is to the degree of market segmentation. Fourth, we study the role of nominal rigidities.

The first two robustness check has obvious policy interest and implications. The motivation for the other two exercises is that the financial crisis may have introduced a (possibly temporary) change in regime, both in the financial market structure and in the price setting mechanism. Ideally, we could capture these phenomena with a regime-switching model. Beside the technical complications, the main limitation of this approach is that the change in regime is probably one of a kind and occurred at the very end of the sample. As such, regime-switching techniques may not have enough data to identify the change in the economic environment. The less formal robustness analysis presented here is still informative to document this point, while further research on this is left for future work.

G.1. *The Role of the Length of LSAP Programmes*

In our baseline simulation, the central bank accumulates assets over four quarters and holds the balance sheet constant for the next two years, before gradually winding down the programme over two additional years. This assumption is fairly arbitrary. Depending on the economic conditions, policy makers may change the length of the programmes, as the recent US and UK experience suggests. Without undertaking an exhaustive analysis, this subsection considers one alternative path: the central bank still accumulates assets over the first year (as per the FOMC announcement in November 2010) but then holds the balance-sheet constant for four years, instead of two, before gradually exiting. Figure G1 shows the corresponding responses, in the same format as the Figures shown in the article, with red continuous line for the this simulation, with grey shaded regions representing the uncertainty and the dashed blue line showing the baseline simulation effects for easy comparison.

Not surprisingly, this change in the time profile of the asset holdings by the central bank induces a stronger response by the risk premium, with a median peak response of -16 bp (instead of -11 bp). As a result, output and inflation respond more strongly. However, while the inflation response roughly doubles compared with the baseline scenario (median response at the peak of 0.059%, compared to 0.031%), the response of output is only 50% stronger (median response of 0.19%, instead of 0.13%, for GDP growth). Not surprisingly, the uncertainty around the median is larger, with the 95th percentiles increasing proportionally.

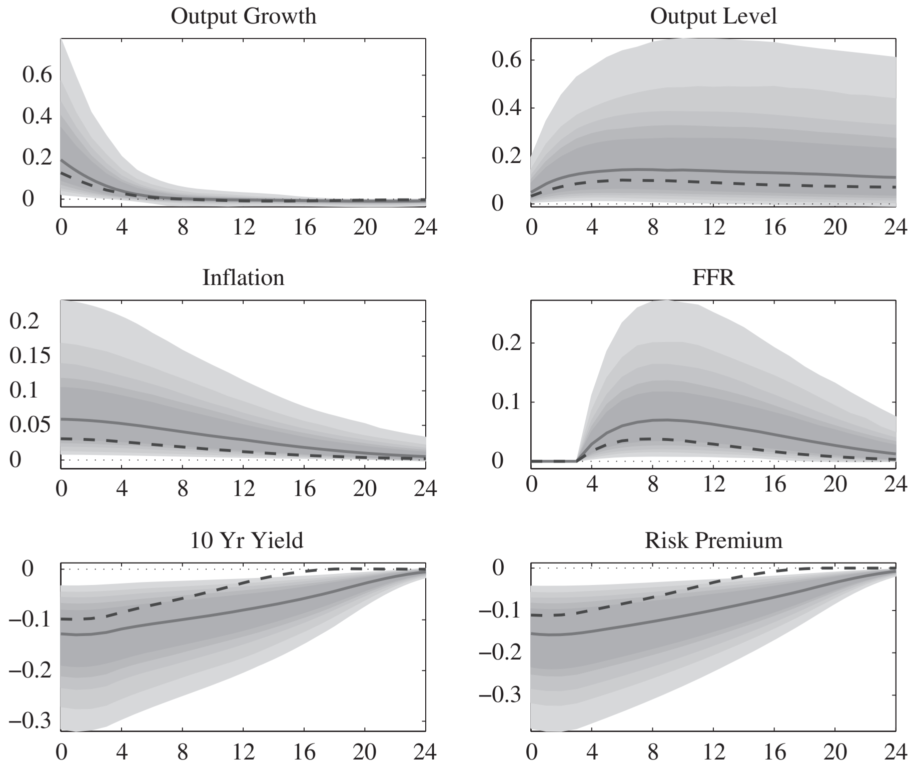

Fig. G1. *Responses to Simulated Shock to Market Value of Long-term Debt (As Shown in Figure 1) in the Case in Which the Central Bank Keeps the Purchased Assets for Four Years (Instead of Two)*
Note. All Responses are in Annualised Percentage Rates (Except the Output Level, Shown in Percentage Deviations from the Path in the Absence of the Shock). The Continuous Line Corresponds to the Posterior Median Response and the Grey Shades to Different Posterior Probability Intervals (50%, 60%, 70%, 80% and 90%, from Darker to Lighter Shading). The Dashed Line is the Posterior Median Response of the Variables in the Baseline Simulation, Shown in Figure 3.

In sum, if the central bank holds the purchased assets for longer, the stimulative impact on output and inflation increases and becomes more persistent. Moreover, the additional boost is stronger for inflation than for output. Nominal rigidities play an important role in this respect. Because the shock lasts longer, more firms and workers are expected to change their prices and wages over time, which in turn leads the firms and workers who can change their prices and wages early to do so more aggressively.

G.2. *The Role of the Length of the ZLB Commitment*

In the article, we discuss how important is the commitment of the central bank to keep the interest rate at zero to boost the effects of the LSAP programme. Here, we take that analysis one step further by considering a longer commitment. Instead of four quarters, we consider five quarters of commitment. Figure G2 shows the corresponding responses, in the same format as Figure G1.

Figure G2 gives a strong message: adding just one more quarter to the commitment gives a powerful boost to the effects of LSAP. GDP growth increases on impact by 0.22% (instead of 0.13%) and inflation increases by 0.045% instead of 0.031. So the effects on the real economy are

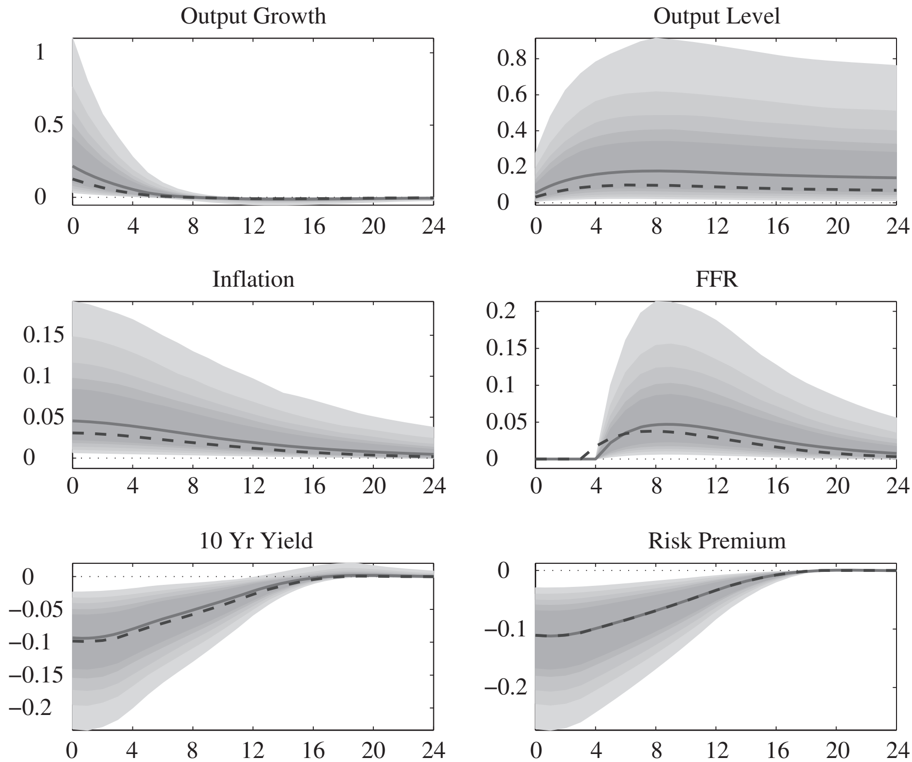

Fig. G2. *Responses to Simulated Shock to Market Value of Long-term Debt* (*Shown in Figure 1*) *in the Case in which the Central Bank Keeps ZLB for Five Quarters* (*Instead of Four*)
Note. All Responses are in Annualised Percentage Rates (Except the Output Level, Shown in Percentage Deviations from the Path in the Absence of the Shock). The Continuous Line Corresponds to the Posterior Median Response and the Grey Shades to Different Posterior Probability Intervals (50%, 60%, 70%, 80% and 90%, from Darker to Lighter Shading). The Dashed Line is the Posterior Median Response of the Variables in the Baseline Simulation, Shown in Figure 3.

stronger by a between 50% and 70%, depending on the variable considered. This means that the effects of additional quarters of commitment to the ZLB are highly non-linear, due to the power of the expectations channel. As a result skewness also increases.

This result also confirms the importance of looking at the effects of interest rate policy and asset purchases in combination. They interact with each other and thus can and should be used in a coordinated fashion.

G.3. *The Role of Market Segmentation*

The baseline experiment suggests that the effects of LSAP II are fairly modest on GDP and quite small on inflation. One reason why our results may underestimate the effects of asset purchase programmes is that the degree of financial market segmentation may have recently increased due to the financial crisis.[2] As discussed earlier, our reduced-form friction for market segmentation aims at

[2] Baumeister and Benati (2010) estimate a VAR with time-varying coefficients and stochastic volatility to account for this type of effects, on top of other changes in the structural relations among macroeconomic variables potentially triggered by the financial crisis.

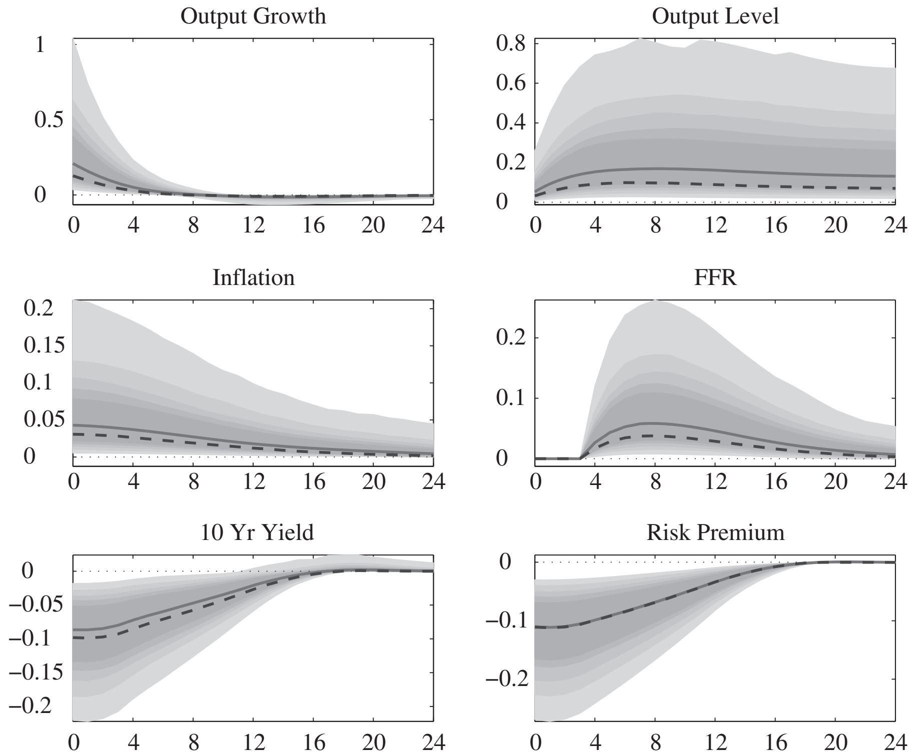

Fig. G3. *Responses to Simulated Shock to Market Value of Long-term Debt (Shown in Figure 1) in the Presence of a High Degree of Market Segmentation (By Considering the Lower Half of the Distribution of ω_u)*
Note. All Responses are in Annualised Percentage Rates (Except the Output Level, Shown in Percentage Deviations from the Path in the Absence of the Shock). The Continuous Line Corresponds to the Posterior Median Response and the Grey Shades to Different Posterior Probability Intervals (50%, 60%, 70%, 80% and 90%, from Darker to Lighter Shading). The Dashed Line is the Posterior Median Response of the Variables in the Baseline Simulation, Shown in Figure 3.

capturing a combination of preferences for certain asset classes and institutional restrictions on the type of investments certain financial intermediaries undertake. By shifting the true and perceived distribution of risk, the financial crisis may have induced a fraction of investors previously active in multiple segments of financial markets to concentrate on one particular asset class.

For this purpose, we repeat the baseline experiment in the presence of a high degree of market segmentation. Figure G3 shows the results of the same simulated LSAP II experiment as in the baseline case. The difference is that, in this case, we only draw from the lower half of the posterior distribution of the parameter ω_u. All other parameters are drawn from the same posterior distribution as before.[3]

The median responses of GDP growth and inflation with the ZLB commitment are about 50% bigger than in the baseline case, at +0.21% and +0.044% respectively (compared with 0.13% and 0.031%). Upside posterior uncertainty is now more pronounced. The 95th percentile now nearly reaches 1% for GDP growth and 0.2% for inflation, compared to 0.6% and 0.15% before.

[3] To be precise, this is a counterfactual simulation. As for the main simulations, we draw a parameter vector from the MCMC posterior sample. However, we then perform a resample exercise for the ω_u parameter in which we extract the marginal sample for this parameter, perform an ascending ordering and keep only the lower half of it. Then, for each parameter vector used in the simulation, we draw independently the ω_u parameter value from this modified subsample.

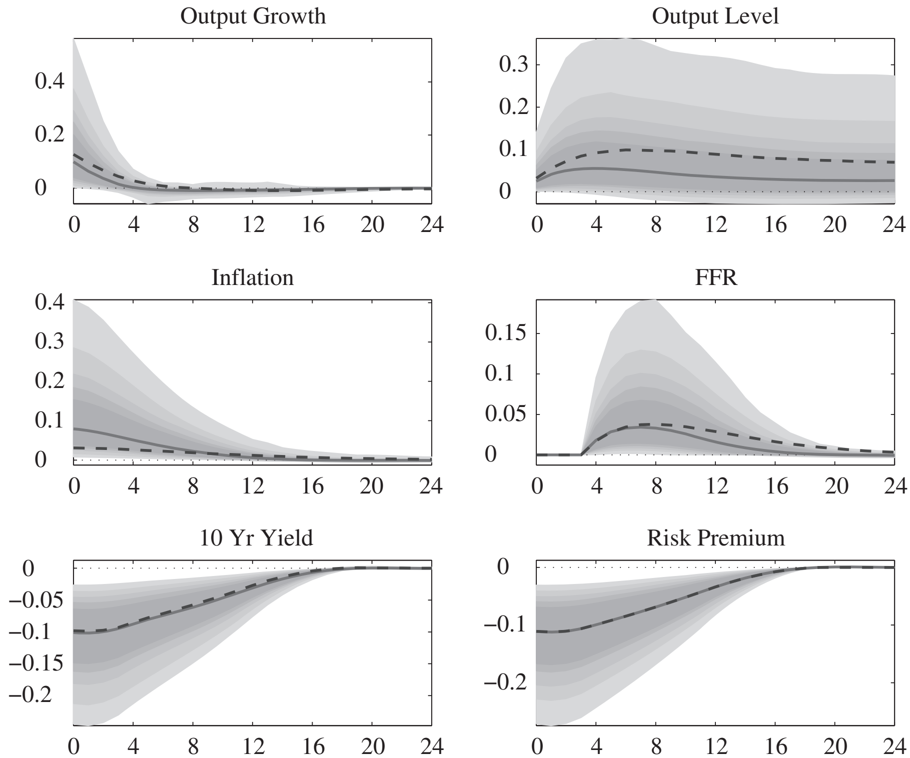

Fig. G4. *Responses to Simulated Shock to Market Value of Long-term Debt (Shown in Figure 1) in the Presence of Lower Price Rigidities* ($\zeta_p = 0.75$)
Note. All Responses are in Annualised Percentage Rates (Except the Output Level, Shown in Percentage Deviations from the Path in the Absence of the Shock). The Continuous Line Corresponds to the Posterior Median Response and the Grey Shades to Different Posterior Probability Intervals (50%, 60%, 70%, 80% and 90%, from Darker to Lighter Shading). The Dashed Line is the Posterior Median Response of the Variables in the Baseline Simulation, Shown in Figure 3.

The stronger response of macroeconomic variables requires the central bank to increase the short-term nominal interest rate by two additional basis points. As a consequence, given that the drop in the risk premium is the same, long-term rates decrease one basis point less.

The bottom line from this exercise is that allowing for a higher degree of segmentation does increase the response of GDP growth and inflation to the stimulus of asset purchase programmes. However, unless segmentation becomes really extreme, the macroeconomic effects of LSAP remain quite small, especially for inflation.

G.4. *The Role of Nominal Rigidities*

One reason for the small response of inflation to LSAP II is that the estimated degree of nominal rigidities, especially for prices, is quite high. While our priors for the probability of holding prices and wages fixed in any given period (ζ_p and ζ_w) are both centred at 0.5, the posterior medians for the two parameters are 0.93 and 0.73 respectively.

A high degree of stickiness in prices and wages is not an uncommon finding in the DSGE literature, especially in the absence of real rigidities like in our case (Del Negro and Schorfheide, 2008). In addition, Hall (2011) has recently emphasised how prices have failed to fall substantially in the last recession. As in the case of segmentation, the financial crisis may have caused a structural change in the price setting process that the model interprets as an increase in price rigidities (the same consideration applies to wages).[4]

Nevertheless, we want to quantify the sensitivity of our results to a lower degree of nominal rigidities, more in line with standard values from the empirical literature that uses micro data (e.g. Nakamura and Steinsson, 2008). Figure G4 shows the results of the baseline LSAP II experiment when we fix ζ_p at 0.75, more in line with the recent empirical evidence. All other parameters are drawn from the same posterior distribution as before.

The Figure shows that nominal rigidities play an important quantitative role in the response of inflation to asset purchases. When prices are more flexible, the median response of inflation to LSAP II on impact is more than two times bigger than when we use the estimated posterior distribution. The counterparts are a less persistent inflation process and a slightly smaller effect on GDP growth. Notice that the effects on the GDP level are now considerably smaller and less persistent. In equilibrium (i.e. taking into account the endogenous response of monetary policy), the two effects roughly compensate each other. The increase in the short-term interest rates is almost the same in the two cases. Therefore, also the behaviour of long-term rates is very similar. The increase in upside uncertainty for inflation is roughly proportional to the changes in the median. The 95th percentile of the response in inflation is 0.4%, compared to just above 0.15% in the baseline experiment.

In sum, higher price flexibility shifts the adjustment in response to asset purchase programmes from GDP growth to inflation, by making its process more front-loaded.

Appendix H. Diagnostics

This Section provides more detailed analysis of the empirical diagnostics for the model. As stated in the main text of the article, since we include the long-term bond as an observable, switching on all the shocks, we should match the short rate and long rate perfectly. To evaluate how the model performs empirically, we want to compare the model-generated moments with those of the data. In the rest of this Section, we analyse variance, variance decomposition and historical shock decomposition in turn.

H.1. *Variance*

In this Section, we compare the variance of each variable in the data with that predicted by our model. For the model variance we compute for each parameter draw the unconditional variance of the relevant state variable and then take the median across draws. We focus on the short-term interest rate and long-term interest rate for this model diagnostics exercise. The model's unconditional variance for the short rate is 0.44, which is just above half of that observed in the data (0.81) while the model's unconditional variance for the long rate is 0.12, which is only one fourth of that observed in the data (0.47). This model does a decent job in terms of explaining the variance of the ratio of the long bond to the short bond (the data variance is 0.08 and the unconditional model variance is 0.07). This suggests that this model has a limited ability to match properties of the yield curve in the data. However, our main purpose is not to explain yield curve shape or dynamics, rather, we are interested in analysing how changes in the risk premium affect macroeconomy and the monetary transmission mechanism of the Fed's unconventional policy.

[4] Indeed, if we consider a sample that ends before the recent crisis (second quarter of 2007), the posterior median for ζ_p is somewhat smaller

H.2. *Variance Decomposition*

Here, we compare the relative importance of different shocks in determining some of the variables of interest, from an unconditional perspective. Table H1 shows the median percentage contribution of each shock to the unconditional variance of some variables of interest. Figures H1–H3 show the variance decomposition at different forecasting horizons for the federal funds rate (FFR), long yield and slope of the yield curve. Yield curve slope is defined as the difference between the long and short rate $rL_t - r_t$. For the definition of risk premium and the long rate implied by the expectation hypothesis, see Section A.6.

Table H1

Variance Decomposition for Short and Long Rates, Slope of the Yield Curve, Risk Premium Component of the Slope and Expectations Hypothesis Component of the Long Rate. For Each Variable, the Table Shows the Median Marginal Contribution of Each Shock to the Unconditional Variance of that Variable, Shown in Percentage Points

	Short rate	Long rate	Slope	Risk premium	Long rate (EH)
Productivity (ε_z)	4.8	9.4	1.4	0.1	12.2
Markup (ε_λ)	0.2	0	0.2	0	0
Investment (ε_μ)	58.4	21.7	52.3	0.1	31.6
Discount factor (ε_b)	1.8	0.5	1.5	0	0.7
Labour supply (ε_ϕ)	18.8	24	5.6	0.1	30.8
Long bond supply (ε_{BL})	0.1	0.7	0.4	0.6	0.2
Tax (ε_T)	0.0	0	0	0	0
Monetary policy (ε_m)	0.6	1.2	6.9	0	1.6
Risk premium (ε_ζ)	5.7	38.6	29.1	98.8	15.7
Government spending (ε_g)	0.1	0	0	0	0

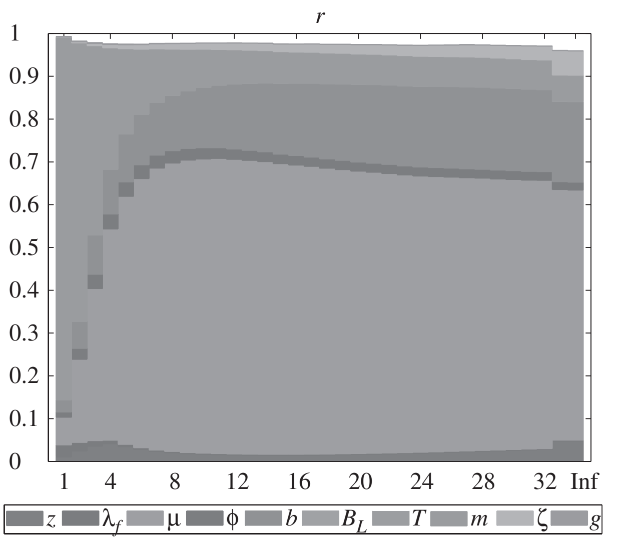

Fig. H1. *Variance Decomposition for the FFR at Different Horizons*

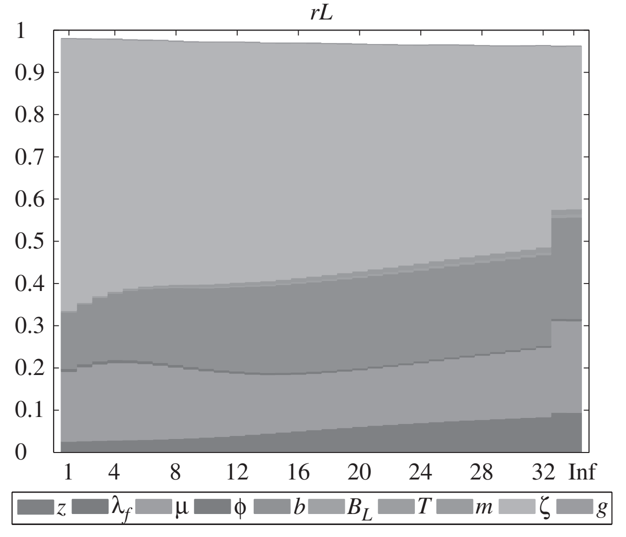

Fig. H2. *Variance Decomposition for the Long Yield at Different Horizons*

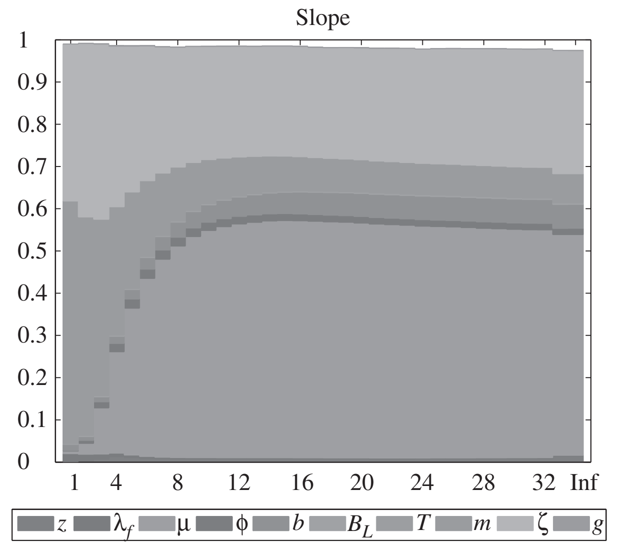

Fig. H3. *Variance Decomposition for the Yield Curve Slope at Different Horizons*

The marginal efficiency of the investment shock (μ) is the single most important factor in determining the short rate at the business cycle frequencies, which is consistent with the findings in Justiniano *et al.* (2010). In the very short run (less than two years), the short rate is also somewhat influenced by the policy shocks. Preference shock (shock to the discount factor) becomes relatively more important to the short rate in the medium to long run, climbing to as much as 19%.

On the contrary, shock to the risk premium is the most important driver for the long yield volatility, accounting for 64% in the short run and 39% in the long run. In second place come the shock to the discount factor (ranges between 13% and 24%) and the shock to the marginal efficiency of investment (ranges between 13% and 22%). Productivity shock accounts for very little in the short run (3%) but rises to 9% in the long run.

The largest contributor to the volatility of the slope of the yield curve is the monetary policy shock. On impact it accounts for 57% and decays to only 7% in the long run. The risk premium shock accounts for as much as 42% two to three quarters ahead, and keeps its important role throughout by fluctuating around 30%, depending on the horizon. The shock to the marginal

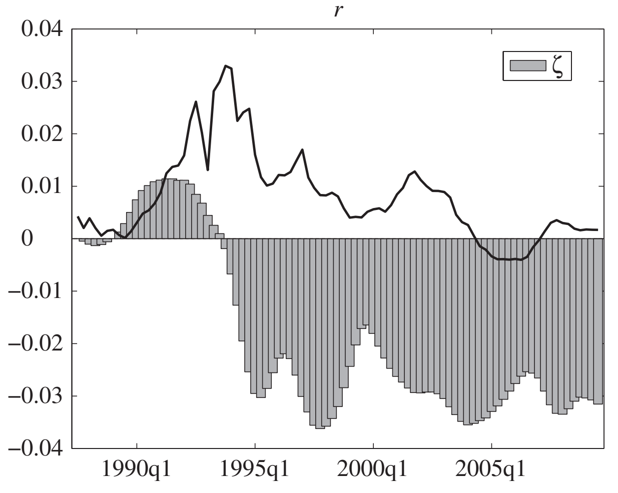

Fig. H4. *Historical Shock Decomposition: Contribution of the Shock to the Risk Premium to Path of the Short Rate*

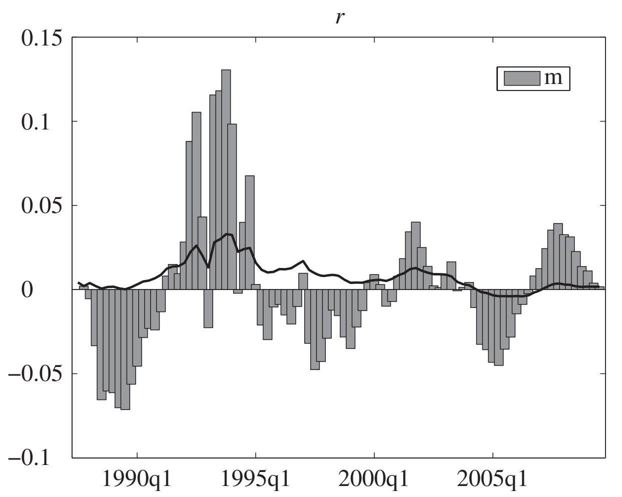

Fig. H5. *Historical Shock Decomposition: Contribution of the Shock to Monetary Policy Rule to the Short Rate* efficiency of investment has a more limited role in the short run but becomes the single most important driver for the volatility of the yield curve slope over the medium and long run, accounting for more than 50% for horizons of eight quarters and longer. Discount factor shock plays a more or less residual role and contributes mostly in the long run, reaching eventually 6%.

If we look at the expectations hypothesis component, then the contributions are similar to the long rate, but now the contribution of the shock to the risk premium is smaller, as expected. The only reason the risk premium shock even shows up here is due to the real effects and the endogenous response of the economy and monetary policy to the shock to the risk premium.

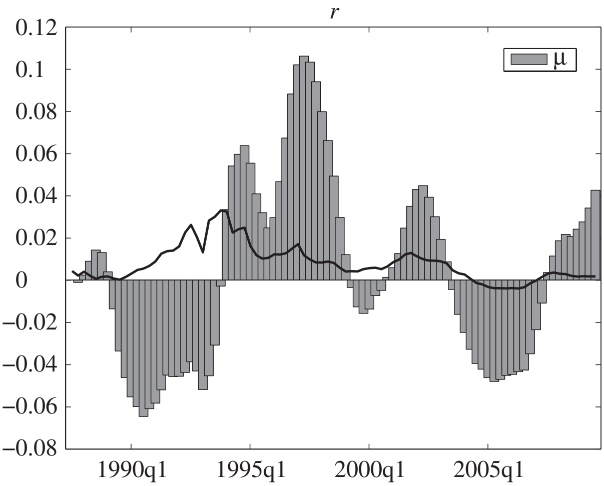

Fig. H6. *Historical Shock Decomposition: Contribution of the Shock to the Marginal Efficiency of Investment to the Path of the Short Rate*

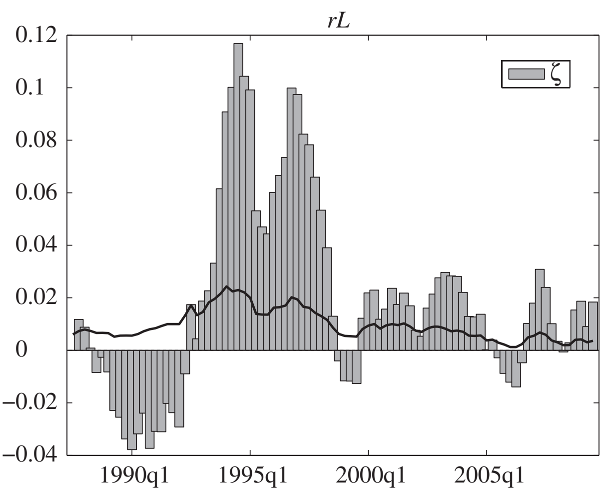

Fig. H7. *Historical Shock Decomposition: Contribution of the Shock to the Risk Premium to the Long Rate*

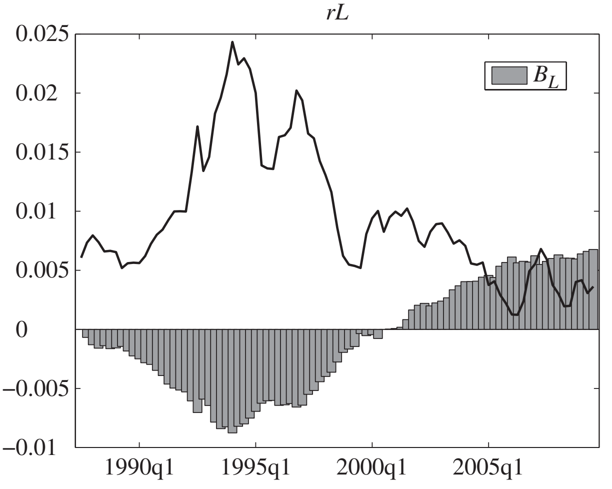

Fig. H8. *Historical Shock Decomposition: Contribution of the Shock to Long Bond Supply to the Path of the Long Rate*

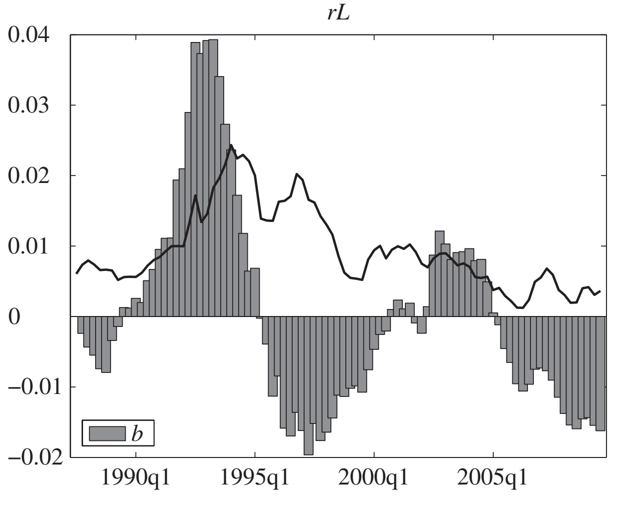

Fig. H9. *Historical Shock Decomposition: Contribution of the Shock to the Discount Factor to the Path of the Long Rate*

H.3. *Historical Shock Decomposition*

In this exercise, we use a disturbance smoother (as described in Carter and Kohn, 1994) to recover draws for the historical paths of the shocks. We then feed these shocks to the model, one at a time, to generate the counterfactual path of each variable, which gives us the marginal contribution of each shock to the evolution of each variable at each point in the sample. We show the median across parameter draws. Figures H4–H14 show select contributions of shocks to the yield curve-related variables. The black line shows the median estimated path for the variable under consideration and the vertical bars show the marginal contribution of each shock in each period in time to that variable's path.

In terms of the short rate, Figure H4 shows that shock to the risk premium has been pushing the FFR down since 1994 by 2–3% points. Interestingly Figure H5 demonstrates that monetary policy shock has been pushing the FFR up since 2007. This means that the recent low interest rates are more likely to be explained by the economic conditions, as opposed to being artificially low due to discretionary policy decisions. As Figure H6 shows, the marginal efficiency of

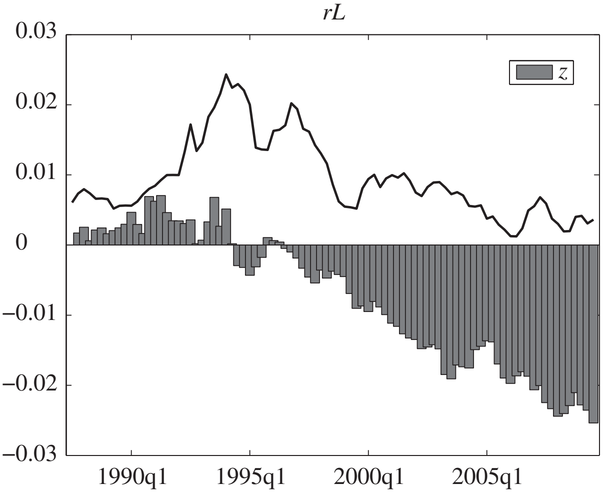

Fig. H10. *Historical Shock Decomposition: Contribution of the Productivity Shock to the Path of the Long Rate*

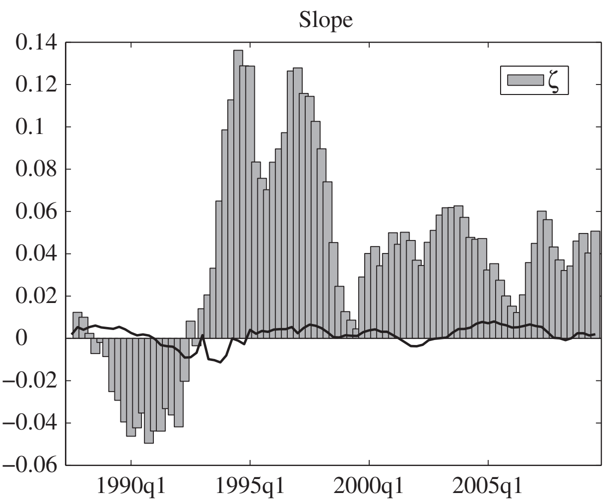

Fig. H11. *Historical Shock Decomposition: Contribution of the Shock to Risk Premium to the Path of the Slope of the Yield Curve*

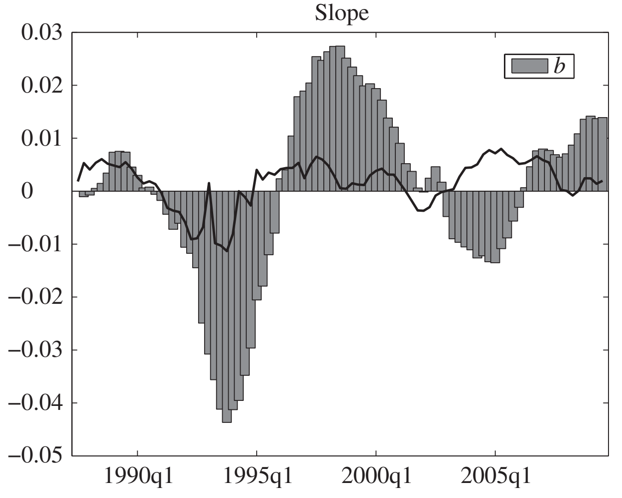

Fig. H12. *Historical Shock Decomposition: Contribution of the Shock to the Discount Factor to the Path of the Slope of the Yield Curve*

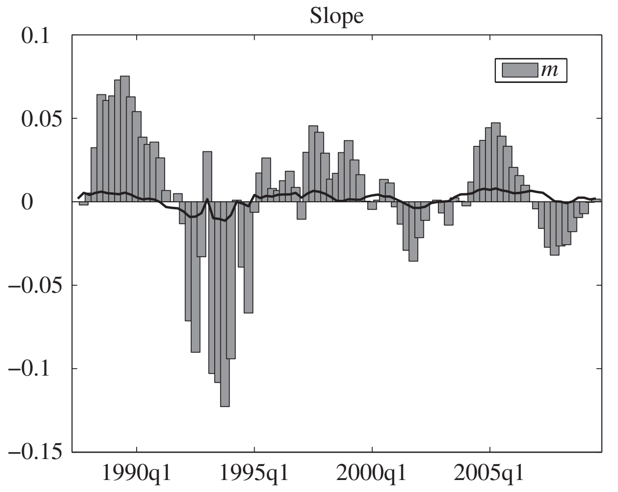

Fig. H13. *Historical Shock Decomposition: Contribution of the Shock to Monetary Policy to the Path of the Slope of the Yield Curve*

investment, the key factor in the unconditional analysis, captures fairly well the cyclical movements in the FFR except the early 90s and the recent period of time. Finally, the productivity shock has been pushing the FFR down since the beginning of 2000.

In the 1990s the risk premium shock (Figure H7) contributes heavily to the movements in the long rate and was compensated down by other shocks. In the most recent period leading to 2009, the risk premium contributes to increase in the long rate, with help from the increasing ratio of long-term debt in the hands of the public. (Also see Figure H8 for the long-term bond supply shock.) On the other hand, Figure H9 shows that the shock to the discount factor has been pushing the long rate down at the end of the sample. Similarly Figure H10 demonstrates the productivity has been pushing down the long yield.

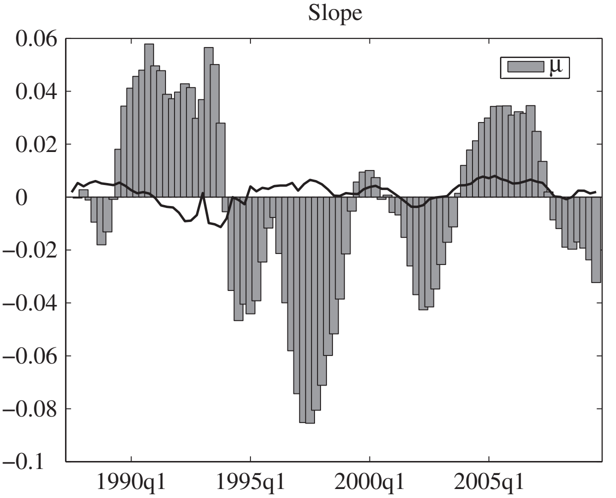

Fig. H14. *Historical Shock Decomposition: Contribution of the Shock to Marginal Efficiency of Investment the Slope of the Yield Curve*

The volatility of the slope of the yield curve is mostly explained by the evolution of the risk premium shock (Figure H11). At the end of the sample, the risk premium shock is pushing the slope up, helped a bit by the shock to the discount factor and the shock to the long-term bond supply that increases the ratio of the long debt to the short debt in the hands of the public. However, the effects are countered by the negative contributions by the monetary policy shock and the shock to the marginal efficiency of investment (See Figures H12–H14).

References

Carter, C.K. and Kohn, R. (1994). 'On Gibbs sampling for state space models', *Biometrika*, vol.81, pp. 541–53.
Hall, R. (2011). 'The long slump', *American Economic Review*, vol. 101, pp. 431–69.
Sims, C.A. (2002). 'Solving linear rational expectations models', *Computational Economics*, vol.20(1–2), pp. 1–20.