Numerical Solution

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1 Problem

The equilibrium is characterized by the following system of dynamic equations , and I solve for the variables $k_{t+1}, b_{t+1}, d_t, r_t^b, R_i, p_{it}$, and q_{it}, A.

$$q_{is} = Ap_{it}^{-\sigma}$$
$$A = \frac{1}{P_{it}^{1-\sigma}}$$
$$P_{it} = \int_0^\infty p_{it}(\omega)d\omega$$

$$\mathrm{p}_{it} = \left(\tfrac{\sigma}{\sigma-1}\right)\left[\tfrac{\tau(r_t^b+\delta)}{Z_t\theta}\right]$$

$$\mathrm{R}_{it}(Z,\theta,r_t^b) = A\left((\tfrac{\sigma}{\sigma-1})[\tfrac{\tau(r_t^b+\delta)}{Z_t\theta}]\right)^{1-\sigma}$$

$$\mathrm{r}_t^b = \frac{1+r-\eta\times(\frac{k_{t+1}}{b_{t+1}})}{1-\eta} - 1$$

The variable Z_t is exogenous and follows the following form (i.e. AR(1) process)

$$\ln z_t = \rho_z \ln z_{t-1} + \sigma_z\epsilon$$

$$R_{it} + [1-(r_t^b+\delta)\tau]k_t] + \frac{b_{t+1}}{1+r_t^b} = k_{t+1} + b_t + \psi(d_t) + f_x$$

$$\frac{1}{1+r}E(\frac{[1-r_{t+1}^b-\delta]\psi_{d_t}(d_t)}{\psi_{dt+1}(d_{t+1})}) = \left[\{A[(\frac{\sigma}{\sigma-1})\frac{\tau}{Z_t\theta}]^{1-\sigma}(1-\sigma)(\frac{1+r-\eta\times(\frac{k_{t+1}}{b_{t+1}})}{1-\eta}-1+\delta)^{-\sigma} - \tau k_t + b_{t+1}\}\frac{\eta}{1-\eta}\frac{k_{t+1}}{(b_{t+1})^2} - 1\right] \tag{1}$$

Note: the in the RHS there is r_{t+1}^b which depends on K_{t+2} and b_{t+2}.

$$\frac{1}{1+r}E(\frac{\psi_{d_t}(d_t)}{\psi_{dt+1}(d_{t+1})}) = \left[\left\{A[(\frac{\sigma}{\sigma-1})\frac{\tau}{Z_t\theta}]^{1-\sigma}(1-\sigma)(\frac{1+r-\eta\times(\frac{k_{t+1}}{b_{t+1}})}{1-\eta}-1+\delta)^{-\sigma} - \tau k_t + b_{t+1}\right\}\frac{\eta}{1-\eta}\frac{k_{t+1}}{(b_{t+1})^2} + \frac{1-\eta}{1+r-\eta\times(\frac{k_{t+1}}{b_{t+1}})}\right] \tag{2}$$

I defined for simplicity in notation $\psi(d_t) = d_t + \kappa(d_t - \bar{d})^2$ where $\bar{d}$ is the steady state (Equilibrium) value . and I also defined defining $\frac{\partial\psi(d_t)}{\partial d_t} = \psi_{dt}(d_t)$ and $\frac{\partial\psi(d_{t+1})}{\partial d_{t+1}} = \psi_{dt+1}(d_{t+1})$.

Table 1: Parameters and Sources

Parameter	**Description**	**Value**	**Source**
σ	Elasticity of Substitution between varieties	4	Constantini and Melitz (2007)
τ	Trade cost	1.35	Constantini and Melitz (2007)
δ	Depreciation rate	0.025	Standard RBC
ρ_z	Agg. shock persistence	0.8857	Begenau and Salomao (2016)
σ_z	Agg. shock std	0.0093	Begenau and Salomao (2016)
r	Risk free rate	0.02	Standard RBC
f_x	Fixed cost of trade	10	Constantini and Melitz (2007)
η	Probability of firm's default	0.05	European Data
θ	Firm's productivity parameter	0.36	Jermann and Quadrini (2012)
κ	Payout fuction parameter	0.1460	Jermann and Quadrini (2012)

Requirement:

I do need to have at the end to have the optimal path of the variables over time.

Moreover, the I need the impulse response funtions of the k and b. (i.e. for a change in the η parameter how would the system dynamic goes towards the new equilibrium and its new equilibrium value.)

Note: at the end there will be an additional requirement that is computing an intergral given the results deriven from the above system. (To be given to the programmer)