Annex Log-Linearizations

I log-linearize the following expression (no-arbitrage condition between bonds and assets for country i):

$$\frac{R_{i,t}}{E_t(\pi_{i,t+1})} = E_t \left\{ \frac{z_{i,t+1} + q_{i,t+1}\left(1 - \delta_i + \frac{a_1 a_3}{1-a_3}\left(\frac{I_{i,t}}{K_{i,t}}\right)^{1-a_3} + a_2\right)}{q_{i,t}} \right\} \tag{1}$$

In steady state I have the following relationship:

$$K_{i,SS} = (1-\delta)K_{i,SS} + I_{i,SS} \tag{2}$$

That implies directly $\delta = \left(\frac{I_{i,SS}}{K_{i,SS}}\right)$. Furthermore, under capital adjustment costs:

$$I_{i,SS} = \left(\frac{a_1 SS}{1 - a_3 SS}\left(\frac{I_{i,SS}}{K_{i,SS}}\right)^{1-a_3 SS} + a_2 SS\right) K_{i,SS} \tag{3}$$

Therefore, I have

$$a_2 SS = \frac{I_{i,SS}}{K_{i,SS}} - \frac{a_1 SS}{1 - a_3 SS}\left(\frac{I_{i,SS}}{K_{i,SS}}\right)^{1-a_3 SS} \tag{4}$$

By definition, $a_3 SS = 1/\xi$. ξ is a parameter < 1.

The corresponding asset pricing equation is:

$$q_{i,t} = \frac{1}{a_1}\left(\frac{I_{i,t}}{K_{i,t}}\right)^{a_3} \tag{5}$$

In steady state, $q_{i,SS} = 1$, therefore I get $a_1 SS = \left(\frac{I_{i,SS}}{K_{i,SS}}\right)^{a_3 SS}$. Moreover, in steady state $\pi_{i,SS} = 1$

I apply now Uhlig's (1999) method to (1), while I substitute the adjustment cost term by $\psi = \frac{a_1 a_3}{1-a_3}\left(\frac{I_{i,t}}{K_{i,t}}\right)^{1-a_3} + a_2$.

Log-linearizing delivers:

$$R_{i,SS}(1 + \widetilde{R_{i,t}} - E(\widetilde{\pi_{i,t+1}})) = (1 + \widetilde{q_{i,t+1}} - \widetilde{q_{i,t}})(1 - \delta + \psi) + (z_{i,SS}))(1 + \widetilde{z_{i,SS}} - \widetilde{q_{i,t}}) \tag{6}$$

Rearranging delivers the well-known linearized form of the NAC condition between bonds and assets as well as an additional term which is multiplied by ψ:

$$\widetilde{R_{i,t}} - E(\widetilde{\pi_{i,t+1}}) = \frac{1-\delta}{R_{i,SS}}(E(\widetilde{q_{i,t+1}})) + E(\widetilde{z_{i,t+1}})\frac{z_{i,SS}}{R_{i,SS}} - \widetilde{q_{i,t}} + \psi\left(\frac{1}{R_{i,SS}}(1 + E(\widetilde{q_{i,t+1}}) - \widetilde{q_{i,t}}))\right) \tag{7}$$

ψ needs also to be log-linearized after the schema $\psi = \psi_{SS} \cdot (e^{\widetilde{\psi}})$. When I plug in all the above steady state expressions for $\psi_{SS} = \frac{a_1 SS a_3 SS}{1-a_3 SS}\left(\frac{I_{i,SS}}{K_{i,SS}}\right)^{1-a_3 SS} + a_2 SS$, I get $\psi_{SS} = 0$. This implies that the adjustment cost derivative term vanishes when I apply log-linearization, is this correct?

My next issue is to log-linearize the asset pricing equation itself:

$$q_{i,t} = \frac{1}{a_1}\left(\frac{I_{i,t}}{K_{i,t}}\right)^{a_3} \tag{8}$$