Modeling the Liquidity Effect: The Limited Participation Model

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Abstract

A tractable limited participation model is developed in order to demonstrate the liquidity effect on interest rates and output. It is also shown that this model can replicate two features of the U.S. economy's response to a positive money shock: an increase in output and a muted response to nominal prices.

1 Introduction

A long standing feature of traditional macroeconomic analysis has been the liquidity effect produced through an expansionary open market operation. As reflected in the upward sloping LM curve, the inverse relationship between the money supply and nominal interest rates has played a critical role in the monetary transmission mechanism and analyses of monetary policy. However, recent modeling developments have begun to minimize the role of the liquidity effect. First and foremost of these is the characterization of monetary policy in terms of an interest rate rule, most notably the Taylor rule. This treatment, in which the money supply is endogenous, permits the modeling of money demand to be placed in the background since implications for the money stock are not the center of analysis.[1]

However, the minimized role of a monetary aggregate (and, hence, money demand) has recently come under question. For instance, Leeper and Roush (2003) demonstrate that the economy's response to a monetary policy shock as estimated from a VAR can be significantly altered by the presence of money. Also, Sims and Zha (2006) show that the introduction of money into an otherwise standard Taylor rule improves their empirical model's coherence with

[1] Another critical development that has reduced interest in models of money demand is the progress that has been seen in estimated stochastic models that minimize monetary elements such as Smets and Wouters (2007).

the data, especially so in those periods such as the initial Volcker era when monetary aggregates played a significant role in the conduct of policy. More recently, Christiano, Mostagno, and Motto (2007) present two theoretical examples in which money can help to anchor inflationary expectations and help to alleviate boom-bust cycles in financial markets. Consequently, while Woodford (2007) argues in favor of policy analysis that discard monetary elements entirely, these empirical and theoretical considerations suggest that such arguments are premature.

With that motivation, I present a tractable general equilibrium model which has the liquidity effect front and center. This model is a variant of a relatively broad class of monetary models referred to as limited participation models which represent a significant extension over the ad hoc money demand representation behind the LM curve. Moreover, this approach attempts to model the flow of funds seriously; this dimension is often ignored in models that place real balances in the utility function. The defining characteristic of this type of model is that not all agents in the economy participate in the monetary process. In particular, new money, in the model presented below, is injected into the banking system rather than distributed among households as a lump-sum transfer. Banks, in turn, lend this new money out to firms; in this way, the effect of an open market operation on interest rates and economic activity are captured. An early version of the limited participation framework was first formally introduced by Grossman and Weiss (1983); a more tractable framework (which exploited a representative agent framework) was developed by Lucas (1990) and Fuerst (1992). More recently, Christiano, Eichenbaum and Evans (1997) explored the quantitative implications of the model and demonstrated that it replicated many features of the U.S. economy's response to a monetary shock.

2 A Simple Limited Participation Model

The model developed below is very closely related to that in Christiano, Eichenbaum and Evans (1997). But, due to some simplifying assumptions, most notably the assumption that money growth is independently distributed over time, I can obtain an analytical solution. This solution, however, captures most of the equilibrium characteristics in the richer model. As mentioned above, a key characterization of the model presented here is that new money is introduced into the banking sector. Another defining attribute of the model is that households must make their savings decisions, i.e. the amount of money they choose to place in the banking sector, before they know the current monetary growth rate (the only source of uncertainty in the model). As this implies, the timing of decisions and the flow of funds are critical in understanding this model. To facilitate that, the timing of events within a time period is listed below:

1. Agents determine how much of their beginning-of-period money, M_{t-1}, they will invest in the banking sector. This investment is denoted I_t and returns the (gross) interest rate, R_t.

2. The current monetary growth rate, γ_t, is known. The implied monetary transfer, X_t, is injected into the banking system. Banks, facing zero costs, lend their total funds, $X_t + I_t$, to firms (at interest rate R_t) so they can pay their wage bill in advance of production.

3. The goods market clears: Firms hire labor in order to produce output and households purchase consumption. Consumption purchases are subject to a cash-in-advance constraint.

4. Firms pay back their loans, banks return the interest on households' investment and the profits due to loans financed the monetary transfer. This determines the money holdings for next period, M_t.

With this overview, I now turn to the various sectors of the economy.

2.1 Firms

Labor, h_t, is the only factor of production and it is assumed that the production function is linear:

$$y_t = \alpha h_t \tag{1}$$

where y_t denotes output. For convenience, the marginal product of labor, α, is assumed to be constant. Since, as noted above, firms finance their labor costs by borrowing at the interest rate, R_t, their profit maximization problem can be expressed as

$$\max_{h_t} \left[P_t y_t - R_t W_t h_t \right] \tag{2}$$

Due to the production function, eq.(1), this yields the familiar condition:

$$R_t \frac{W_t}{P_t} = \alpha \tag{3}$$

That is, the marginal cost of production inclusive of financing costs must, at the firm's optimum, be equal to the (constant) marginal product of labor. Note that this has the immediate implication that interest rates and real wages will be inversely related.

2.1.1 Central Bank

The central bank's only role is to provide new money into the economy. The law of motion of the money stock is given by:

$$M_t = (1 + \gamma_t) M_{t-1} \tag{4}$$

It is assumed that γ_t is a random variable that follows a stationary Markov process with transitions governed by:

$$G(\gamma', \gamma) = \Pr \left[\gamma_{+1} < \gamma' \,|\gamma_t = \gamma \right] \tag{5}$$

While we keep this general form for now, in the characterization of equilibrium it will be assumed that the money growth process is *i.i.d.*

2.2 Banks

Banks have no costs and, consequently, inelastically supply all of their available funds to firms in the loan market. Hence, we have:

$$I_t + X_t = W_t h_t \tag{6}$$

2.3 Households

Households maximize expected lifetime utility given by:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln c_t - \frac{\psi}{1+\eta} h_t^{1+\eta} \right] \right\} \tag{7}$$

subject to their budget constraint:

$$M_t = (M_{t-1} - I_t) - P_t c_t + W_t h_t + I_t R_t + X_t R_t \tag{8}$$

Note the last two terms represent payments from the bank in the form of interest on deposits and profits due to the monetary injection. After placing their savings in the banking system and receiving payment in advance of their labor efforts, households face a cash-in-advance constraint on consumption purchases:

$$(M_{t-1} - I_t) + W_t h_t \geq P_t c_t \tag{9}$$

It is useful to note, in anticipation of equilibrium, that there are two nominal magnitudes in this economy: the total labor costs ($W_t h_t$) which determines the amount of loans in the economy, and the nominal value of expenditures, $P_t c_t$. The relative size of loans and purchases helps to determine equilibrium behavior in this model.

This maximization problem can be expressed as a dynamic programming problem and the necessary conditions derived formally. This is presented in the Appendix. Here, however, I motivate the conditions describing agents' optimal labor-leisure and consumption-savings choices intuitively.

The labor-leisure choice is represented by the following:

$$\psi h_t^{\eta} = \frac{1}{c_t} \frac{W_t}{P_t} \tag{10}$$

This is the standard condition in which the marginal disutility of work is equal to the marginal benefit due to increased consumption financed by work activity.[2] Note that $(1/\eta)$ is the elasticity of labor supply.

[2] Note that, unlike most cash-in-advance models, there is no inflation tax associated with labor income. This is due to the assumption that workers are paid in advance of their production. In a critical appraisal of the limited participation model, Auray and Fève (2005) did not make this key assumption so that, in their model, the inflation tax outweighed the liquidity effect. Consequently, interest rates were positively related to monetary growth.

The necessary condition associated with the savings decision is not standard due to the assumption that this decision is made before time t information (i.e. the monetary growth rate) is known:

$$E_{t-1}\left[\frac{1}{c_t}\frac{1}{P_t}\right] = E_{t-1}\left[\beta R_t E_t\left[\frac{1}{c_{t+1}}\frac{1}{P_{t+1}}\right]\right] \tag{11}$$

As discussed in Christiano (1991), this condition is the hallmark of this version of the limited participation framework. In a typical model, the choice of savings at time t is made with full information so the time $t-1$ dated expectations are not relevant. In that case, the nominal interest, R_t, reflects the standard Fisherian factors as represented by the nominal marginal rate of substitution. While in this model these Fisherian factors determine the *average* behavior of nominal interest rates, the nominal interest rate will also be affected by liquidity injected into the banking sector. This will be demonstrated below.

3 Definition of Equilibrium

In equilibrium, we require that the goods market clears so that $c_t = y_t = \alpha h_t$. Hence equilibrium is defined in terms of two quantities, labor and savings, and one price - the nominal interest rate, R_t. Once these are determined, all other quantities and prices, e.g. the real wage, can be computed. For a stationary recursive equilibrium, we require that these equilibrium values can be expressed as time-invariant functions of the state. Given that savings decisions are made with time $t-1$ information and that these decisions will influence the nominal interest rate and labor at time t, the state is defined by the vector $\left(\gamma_t, \gamma_{t-1}\right)$. Also, since the money supply is growing over time this implies that I_t will be as well. To make this a stationary variable, I deflate this by the beginning-of-period money stock. That is $i_t \equiv I_t/M_{t-1}$. Note that this variable will be a function of γ_{t-1} only. Hence, equilibrium is defined by the following functions:

$$\begin{aligned} i_t &= i\left(\gamma_{t-1}\right) \\ h_t &= h\left(\gamma_{t-1}, \gamma_t\right) \\ R_t &= R\left(\gamma_{t-1}, \gamma_t\right) \end{aligned}$$

These functions must satisfy the following conditions:

$$\text{Equilibrium in the goods market} \quad : \quad (M_{t-1} - I_t) + W_t h_t = P_t c_t \tag{12}$$

$$\text{Equilibrium in the Loan market} \quad : \quad I_t + X_t = W_t h_t \tag{13}$$

In addition, firms' and households' labor decisions must be optimal as characterized by eqs. (3) and (10) respectively. Also, the necessary condition associated with optimal savings decisions must be satisfied. That is, eq. (11) must hold.

4 Characterization of Equilibrium

To characterize equilibrium, I assume that the monetary growth rate is independently distributed over time. This greatly simplifies the analysis since, as will be demonstrated below, the fraction of money placed into the banking system, $i\left(\gamma_{t-1}\right)$, is constant. But first note that equilibrium in the loan and goods markets implies:

$$P_t c_t = M_{t-1} + X_t = M_t \tag{14}$$

That is, the money stock is equal to the nominal value of consumption - a standard result in CIA models. To solve for the equilibrium interest rate, note that eq. (3) can be written as:

$$R\left(\gamma_{t-1}, \gamma_t\right) = \alpha \frac{P_t}{W_t} = \alpha \frac{P_t c_t}{W_t c_t} = \frac{P_t c_t}{W_t h_t} \tag{15}$$

Hence, as stated earlier, the nominal interest rate will reflect the relative liquidity in the goods and loan markets. This can be combined with the equilibrium conditions in both markets to yield:

$$R\left(\gamma_{t-1}, \gamma_t\right) = \frac{M_t}{I_t + X_t} = \frac{M_{t-1}\left(1+\gamma_t\right)}{M_{t-1}\left(i\left(\gamma_{t-1}\right)+\gamma_t\right)} = \frac{\left(1+\gamma_t\right)}{\left(i\left(\gamma_{t-1}\right)+\gamma_t\right)} \tag{16}$$

As can be seen in the above expression, the nominal interest rate will, in general be a function of both $t-1$ and t monetary growth rates. But, more importantly, since $i\left(\gamma_{t-1}\right)$ is predetermined and less than one, we have immediately that the nominal interest rate will vary inversely with the monetary growth rate. A liquidity effect will be present in this model since a monetary injection places relatively more liquidity in the banking (i.e. loan sector) than in the goods market; as banks seek to loan out these additional funds, the interest rate must fall to clear the loan market.

We use this result to characterize investment decisions in the *i.i.d.* framework. Note that, again using the goods market equilibrium condition, the savings optimality condition (eq. (11)) can be written as:

$$E_{t-1}\left[\left(1+\gamma_t\right)^{-1}\right] = E_{t-1}\left[R\left(\gamma_{t-1}, \gamma_t\right)\left(1+\gamma_t\right)^{-1} \beta E_t\left[\left(1+\gamma_{t+1}\right)^{-1}\right]\right] \tag{17}$$

Then using the expression for the nominal interest rate in eq. (16) and the fact that, under independent growth rates, the forecast of next period's monetary growth rate is constant, we have (where time-subscripts have been dropped from the expectations operator because of the *i.i.d.* assumption):

$$\beta^{-1} = E\left[\frac{1}{i\left(\gamma_{t-1}\right)+\gamma_t}\right] \tag{18}$$

This implicitly defines a unique $\bar{\imath} = i\left(\gamma_{t-1}\right)$.[3] Given that the monetary growth

[3] The proof is straightforward and uses the uniqueness of equilibrium. If $\bar{\imath}$ is constant, then γ_t can be expressed as $\gamma_t = \bar{\imath} k\left(\gamma_t\right)$ where $k\left(\gamma_t\right)$ is simply $\gamma_t/\bar{\imath}$. Then $\bar{\imath}$ can be factored out of the expectations operator and, by the *i.i.d.* assumption, the expectations term is constant. So this implies the original conjecture of a constant $\bar{\imath}$ is verified.

rate provides no information about the future, it is not surprising that households save a constant fraction of their beginning-of-period nominal balances. Note that, from eq. (16), this implies that the nominal interest rate in this setting will be a function of the current monetary growth rate only.

Finally, turning to equilibrium labor we can combine the firm's and household's optimality conditions to yield:

$$c_t \psi h(\gamma_t)^\eta = \frac{W_t}{P_t} = \alpha R(\gamma_t)^{-1}$$

Or, using the production function and re-arranging terms:

$$h(\gamma_t) = AR(\gamma_t)^{-\frac{1}{1+\eta}} \tag{19}$$

where $A = \psi^{1/(1+\eta)}$. This demonstrates that labor (and therefore output) will be positively related to the monetary growth rate via its effect on the nominal interest rate. Hence this model will capture the traditional monetary transmission mechanism: a positive monetary growth rate will cause nominal interest rates to fall. The resulting decline in financing costs to firms will stimulate the demand for labor and, consequently, output will increase. Note also that the economy is moving along a fixed labor supply curve so real wages will also be procyclical.

If the labor supply elasticity is great enough, the increased output can, in principle, entirely offset the inflationary pressures due to money growth. From the CIA constraint, we have that the inflation rate is:

$$1 + \pi_t = \frac{P_t}{P_{t-1}} = \frac{M_t}{M_{t-1}} \frac{c_{t-1}}{c_t} = (1 + \gamma_t) \left(\frac{R(\gamma_t)}{R(\gamma_{t-1})} \right)^{\frac{1}{1+\eta}}$$

where the last expression comes from the production function and the result in eq. (19). Hence if the fall in interest rates is large relative to the monetary growth rate and if η is sufficiently small (implying a high elasticity of labor supply) then inflation will be low. Consequently, as pointed out by Christiano, Eichenbaum and Evans (1997), this model can replicate the stylized facts for the U.S. economy in that a monetary shock results in an immediate increase in output but a muted response in prices.

5 Conclusion

The model developed above presents a tractable analysis of the liquidity effect in a stochastic, dynamic general equilibrium setting. Hence, it may be a useful pedagogical tool for those instructors who wish to discuss money demand in a modern context. There are also relatively straightforward extensions to the analysis conducted here. For instance, increases in uncertainty about monetary policy could be analyzed (as done in a slightly richer context in Jordá and Salyer (2003)) via a mean-preserving spread in the monetary growth rate process.

Since, as seen in eq. (18), the expectations is over a convex function of the monetary growth rate, this will have implications for $\bar{\imath}$, i.e. the amount of savings placed in the banking sector.

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6 Appendix

As described above, the household's decision problem involves choices that are made with different information sets. To capture this, it is convenient to express the problem in a slightly different manner. In this modified format, all decisions are made at the same time, but we model the investment decision as being similar to a forward contract. That is, a binding decision is made at time t that has implications for the household's constraints (and therefore utility) in period $t+1$. Hence, in this modified framework, the household's relevant state variables are beginning of period money, M_{t-1}, and the number of forward contracts purchased in the previous period, F_{t-1}. Denote this state vector as $s_t = \left(\frac{M_{t-1}}{P_t}, F_{t-1}\right)$. Each forward contract purchased at time $t-1$ reduces the amount of money that is available for consumption in period t by \$1 but augments income by the amount $\$(R_t - 1)$. With this environment, the agent's maximization problem can be expressed as the following dynamic programming problem (written in the form of a Lagrangian)

$$
\begin{aligned}
V(s_t) &= \max_{(c_t, h_t, M_t, F_t)} \left\{ \ln c_t - \frac{\psi}{1+\eta} h_t^{1+\eta} + \beta E_t \left[V(s_{t+1}) \right] \right\} && (20) \\
&\quad + \lambda_t \left[\frac{M_{t-1}}{P_t} + \frac{F_{t-1}}{P_t}(R_t - 1) + \frac{W_t}{P_t} h_t - c_t - \frac{M_t}{P_t} \right] && (21) \\
&\quad + \mu_t \left[\frac{M_{t-1}}{P_t} - \frac{F_{t-1}}{P_t} + \frac{W_t}{P_t} h_t - c \right] && (22)
\end{aligned}
$$

The necessary conditions associated with this problem are:

$$c_t : \; c_t^{-1} - \lambda_t - \mu_t = 0 \tag{23}$$

$$h_t : \; -\psi h_t^{\eta} + \lambda_t \frac{W_t}{P_t} + \mu_t \frac{W_t}{P_t} = 0 \tag{24}$$

$$M_t : \; \beta E_t \left[\frac{\partial V(s_{t+1})}{\partial (M_t / P_{t+1})} \frac{1}{P_{t+1}} \right] - \lambda_t \frac{1}{P_t} = 0 \tag{25}$$

$$F_t : \; \beta E_t \left[\frac{\partial V(s_{t+1})}{\partial F_t} \right] = 0 \tag{26}$$

To simplify these expressions, use the envelope theorem in order to eliminate the derivatives of the value function:

$$\frac{\partial V(s_t)}{\partial (M_{t-1}/P_t)} = (\lambda_t + \mu_t) \tag{27}$$

$$\frac{\partial V(s_t)}{\partial F_{t-1}} = \frac{\lambda_t}{P_t}(R_t - 1) - \frac{\mu_t}{P_t} \tag{28}$$

Combining eqs. (23) and (24) produces the labor-leisure necessary condition as seen in eq. (10)

$$\psi h_t^\eta = \frac{1}{c_t}\frac{W_t}{P_t} \tag{29}$$

Note that, as implied by eq. (25) when combined with the envelope theorem, the Lagrange multiplier on real wealth at time t is equal to the expected utility gain that a unit of real balances will provide in period $t+1$:

$$\lambda_t = \beta E_t\left[\frac{P_t}{c_{t+1}P_{t+1}}\right] \tag{30}$$

Finally, turning to the necessary condition associated with the forward contract, the envelope theorem implies:

$$E_t\left[\frac{1}{c_{t+1}P_{t+1}}\right] = E_t\left[R_{t+1}\frac{\lambda_{t+1}}{P_{t+1}}\right]$$

But eq. (30) permits the right-hand side to be re-written so that the expression becomes:

$$E_t\left[\frac{1}{c_{t+1}P_{t+1}}\right] = E_t\left[R_{t+1}\beta E_{t+1}\left(\frac{1}{c_{t+2}P_{t+2}}\right)\right]$$

Since I_t in the model of the text corresponds to F_{t-1} in this modified model, the above expression needs to be lagged one period. But this reproduces eq. (11).