1 Model

The Epstein-Zin-Weil objective function is defined recursively as

$$U_t = \left\{ (1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

where ψ is the intertemporal elasticity of substitution in consumption (IES), γ is the coefficient of relative risk aversion, and define $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$.

Consider the consumption process as follows, and the volatility of consumption shocks σ_t^2 follows a AR(1) process:

$$\Delta c_{t+1} = \mu_c + \sigma_t e_{1,t+1} \tag{1}$$

$$\sigma_{t+1}^2 = \mu_\sigma + \rho_\sigma (\sigma_t^2 - \mu_\sigma) + \varphi_\sigma e_{2,t+1} \tag{2}$$

in which Δc_{t+1} is logarithm consumption growth. Both the shocks $e_{1,t+1}$ and $e_{2,t+1}$ are standard Normal.

The logarithm of the intertemporal marginal rate of substitution for recursive preferences is given by

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1} \tag{3}$$

in which $r_{c,t+1}$ is logrithm return on wealth, which pays consumption as dividend at each period. Z_t denotes the wealth-to-consumption ratio (i.e. price-to-dividend ratio).

$$R_{c,t+1} = \frac{W_{t+1} + C_{t+1}}{W_t} = \left(\frac{1 + Z_{t+1}}{Z_t} \right) \frac{C_{t+1}}{C_t}$$

The Euler equation is

$$Z_t = E_t \left[\exp(m_{t+1}) \left((1 + Z_{t+1}) \frac{C_{t+1}}{C_t} \right) \right]$$

2 Model Setup in Dynare

The model in Dynare is specified in exp-logs format, as follows:

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dc = mu_c + sqrt(sigsqr(-1))*e1;
sigsqr = mu_sig + rho_sig*(sigsqr(-1)-mu_sig) + phi_sig*e2;
exp(m) = (beta^theta)*((exp(dc))^(-theta/psi))*((exp(rc))^(theta-1));
exp(z) = exp(m(+1))*(1+exp(z(+1)))*exp(dc(+1));
exp(rc) = (1+exp(z))*exp(dc)/exp(z(-1));
```