DSGE Models: Calibration, Impulse Responses, Simulated Data—part III

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Lecture outline

- RBC model with a stochastic discount factor (cont'd)
- Impulse responses
- The role of the Frisch labour elasticity
- Greenwood Hercowitz Krussel 1988 AER model

RBC PLUS (Frisch elasticity)

- What is the role of ϕ?
- Inverse elasticity of labor supply with respect to the real wage holding constant the marginal utility of consumption.
- Since preferences are separable in consumption and labour supply, denote $U(C_t) = \log(C_t)$ and $\nu(N_t) = \frac{\eta N_t^{1+\phi}}{1+\phi}$
- Note that $U'(C_t) = \frac{1}{C_t}$, $U''(C_t) = \frac{-1}{C_t^2}$, $\nu'(N_t) = \eta N_t^{\phi}$, and $\nu''(N_t) = \eta\phi N_t^{\phi-1}$

RBC PLUS (Frisch elasticity)

- In a decentralized economy,

$$
\begin{aligned}
-B_t \nu'(N_t) + \lambda_t W_t &= 0 && (1)\\
B_t U'(C_t) &= \lambda_t && (2)\\
& && (3)
\end{aligned}
$$

where W_t is the real wage at t. From (1) and (2) we get

$$
\nu'(N_t) - W_t U'(C_t) = 0 \tag{4}
$$

RBC PLUS (Frisch elasticity)

- Log-linearizing (4) around the steady state we get

$$\begin{aligned} \frac{N\nu''}{\nu'} n_t &= \frac{CU''}{U'} c_t + w_t && (5) \\ \phi n_t &= w_t - c_t && (6) \\ n_t &= \frac{1}{\phi} w_t - \frac{c_t}{\phi} && (7) \end{aligned}$$

- ϕ is *the elasticity of the marginal disutility of work with respect to hours worked*
- $\frac{1}{\phi}$ is FRISCH ELASTICITY *of labour supply. It represents the elasticity of labour supply with respect to changes in the current wage rate, keeping fixed the marginal utility of consumption*

RBC PLUS (Frisch elasticity)

- Therefore, ϕ is sometimes referred to as the *inverse* of the elasticity of labour supply
- For separable utility functions, the marginal utility of consumption is proportional to consumption. Not so for non-separable utility function
- If $w_t = 1\%$, current labour supply will rise by $\frac{1}{\phi}\%$, holding constant the marginal utility of consumption. Thus, the smaller the ϕ, the greater the Frisch elasticity, the flatter the labour supply curve, the larger the response of employment.

RBC PLUS (Frisch elasticity)

- In a decentralized economy, the Euler equation for optimal level of capital is

$$\lambda_t = \beta E_t \left[\lambda_{t+1} R_{t+1} \right] \tag{8}$$

where R_{t+1} is the gross return on capital. From (1)

$$\lambda_t = \frac{B_t \nu'(N_t)}{W_t} \tag{9}$$

$$\lambda_{t+1} = \frac{B_{t+1} \nu'(N_{t+1})}{W_{t+1}} \tag{10}$$

RBC PLUS (Frisch elasticity)

- Substitute (9) and (10) in (8) to get

$$\frac{B_t \nu'(N_t)}{W_t} = \beta E_t \left[\frac{B_{t+1}\nu'(N_{t+1})}{W_{t+1}} R_{t+1} \right] \tag{11}$$

  Log-linearizing (11) around the steady state and rearranging terms gives

$$n_t - E_t n_{t+1} = \frac{1}{\phi}(w_t - E_t w_{t+1}) + \frac{1}{\phi}(E_t b_{t+1} - b_t) + E_t r_{t+1} \tag{12}$$

- Equation (12) shows how the FRISCH ELASTICITY is also the intertemporal elasticity of substitution of work (or leisure) with respect to a change in future wage.
- If future wage is expected to be high relative to the current wage, $w_t - E_t w_{t+1} < 0$, then $n_t - E_t n_{t+1} < 0$. That is, the consumer will reduce current labour supply (or increase current leisure) and postpone work to tomorrow.

RBC PLUS (Frisch elasticity)

- Note that if $E_t r_{t+1} > 0$ then $n_t - E_t n_{t+1} > 0$ given other variables. The representative consumer works harder today relative to tomorrow, irrespective of the slope of the labour supply curve.
- A preference shock t implies that $E_t b_{t+1} - b_t < 0$. This shock will cause n_t to fall relative to tomorrow's labour supply. For the larger the ϕ, the smaller the Frisch elasticity, the smaller the fall in current labour.

GHH

- In an important contribution GHH introduce shocks to the marginal efficiency of investment and variable capacity utilization into an (almost) otherwise standard RBC model
- This is the first paper to argue about the importance of shocks in the investment sector of the economy: **investment-specific** shocks
- These are quite different from the TFP shocks as they only affect productivity of “new” capital that comes in line by new investment expenditure
- This model is the precedent for a great body of work that studies investment specific shocks

GHH

- Almost everything is standard in this model apart from:
- The CAE: $K_{t+1} = (1 - \delta(H_t))K_t + i_t(1 + z_t)$
- where, z_t is an investment-specific technology shock, and H_t is the utilization of capital
- Accordingly the production function becomes: $Y_t = (K_t H_t)^{\alpha} N_t^{1-\alpha}$
- Notice there is no TFP shock — as we want to focus on the IS shock

GHH

- The other main difference comes in the specification of preferences
- GHH assume that hours worked are determined independently of the consumptions-savings decision
- In other words the marginal rate of substitution between consumption and hours only depends on hours and not on consumption
- GHH want to focus on the aspect of the labour market that is influenced by the variable rate of utilization rather than the intertemporal substitution channel
- $\frac{1}{1-\gamma}\left[\left(C_t - \frac{N_t^{1+\theta}}{1+\theta}\right)^{1-\gamma} - 1\right]$

GHH

- Set up the Social Planner's problem
- The Social Planner's problem is to choose $\{C_t, N_t, H_t, K_{t+1}, \lambda_t\}$, for $t = 0, 1, ...$ to maximize

$$\max Ł = \max E_0 \sum_{t=0}^{\infty} \beta^t \Big(\frac{1}{1-\gamma} \left[\left(C_t - \frac{N_t^{1+\theta}}{1+\theta} \right)^{1-\gamma} - 1 \right] - \lambda_t \left(C_t + \frac{K_{t+1}}{1+Z_t} - \frac{K_t(1 - \frac{1}{\omega} H_t^{\omega})}{1+Z_t} - (K_t H_t)^{\alpha} N_t^{1-\alpha} \right) \Big) \tag{13}$$

GHH

- The FOCs are:

$$\frac{\partial \text{Ł}}{\partial C_t} = 0 \quad \Leftrightarrow \quad \left(C_t - \frac{N_t^{1+\theta}}{1+\theta} \right)^{-\gamma} - \lambda_t = 0 \tag{14}$$

$$\frac{\partial \text{Ł}}{\partial N_t} = 0 \quad \Leftrightarrow \quad \left(C_t - \frac{N_t^{1+\theta}}{1+\theta} \right)^{-\gamma} (-N_t^{\theta}) + \lambda_t (1-\alpha)(K_t H_t)^{\alpha} N_t^{-\alpha} = 0 \tag{15}$$

$$\frac{\partial \text{Ł}}{\partial H_t} = 0 \quad \Leftrightarrow \quad -\lambda_t \left(\frac{K_t H_t^{\omega-1}}{1+Z_t} - \alpha \left(K_t H_t\right)^{\alpha-1} K_t N_t^{1-\alpha} \right) = 0 \tag{16}$$

$$\frac{\partial \text{Ł}}{\partial K_{t+1}} = 0 \quad \Leftrightarrow \quad \frac{-\lambda_t}{1+Z_t} + \beta E_t \lambda_{t+1} \left(\alpha \left(K_{t+1} H_{t+1}\right)^{\alpha-1} N_{t+1}^{1-\alpha} H_{t+1} + \frac{1 - \frac{1}{\omega} H_{t+1}^{\omega}}{1+Z_{t+1}} \right) = 0 \tag{17}$$

$$\frac{\partial \text{Ł}}{\partial \lambda_t} = 0 \quad \Leftrightarrow \quad C_t + \frac{K_{t+1}}{1+Z_t} - \frac{K_t \left(1 - \frac{1}{\omega} H_t^{\omega}\right)}{1+Z_t} = \left(K_t H_t\right)^{\alpha} N_t^{1-\alpha} \tag{18}$$

GHH

- Notice that λ_t cancels out in (15) and (16), therefore, these conditions are *atemporal*. Together they determine N_t and H_t. The Euler equation (17) and the feasibility constraint (18) determine C_t and K_{t+1}. We can simplify the FOCs and write (after eliminating λ_t)

$$N_t^{\theta} = (1-\alpha)\left(K_t H_t\right)^{\alpha} N_t^{-\alpha} \tag{19}$$

$$\frac{H_t^{\omega-1}}{1+Z_t} = \alpha\left(K_t H_t\right)^{\alpha-1} N_t^{1-\alpha} \tag{20}$$

$$\frac{\left(C_t - \frac{N_t^{1+\theta}}{1+\theta}\right)^{-\gamma}}{1+Z_t} = \beta E_t \left(C_{t+1} - \frac{N_{t+1}^{1+\theta}}{1+\theta}\right)^{-\gamma}$$

$$\left(\left(\alpha\left(K_{t+1}H_{t+1}\right)^{\alpha-1} N_{t+1}^{1-\alpha} H_{t+1} + \frac{1-\frac{1}{\omega}H_{t+1}^{\omega}}{1+Z_{t+1}}\right)\right. \tag{21}$$

$$\left(K_t H_t\right)^{\alpha} N_t^{1-\alpha} = C_t + \frac{K_{t+1}}{1+Z_t} - \frac{K_t\left(1-\frac{1}{\omega}H_t^{\omega}\right)}{1+Z_t} \tag{22}$$

GHH

- The steady state can be computed as follows: Assume that $\log(Z) = 0$ or $Z = 1$. Equations (19) - (22) in the steady state are

$$
\begin{aligned}
N^{\theta} &= (1-\alpha)(KH)^{\alpha} N^{-\alpha} && (23)\\
H^{\omega-1} &= \alpha (KH)^{\alpha-1} N^{1-\alpha} && (24)\\
1 &= \beta\left(\alpha (KH)^{\alpha-1} N^{1-\alpha} H + 1 - \frac{1}{\omega} H^{\omega}\right) && (25)\\
(KH)^{\alpha} N^{1-\alpha} &= C + \frac{1}{\omega} H^{\omega} K && (26)
\end{aligned}
$$

From (25),

$$\frac{1}{H}\left(\frac{1}{\beta} - 1 + \frac{1}{\omega} H^{\omega}\right) = \alpha (KH)^{\alpha-1} N^{1-\alpha} \qquad (27)$$

Plug (27) in (24) and solve for the steady state level of utilization H as

$$
\begin{aligned}
H^{\omega-1} &= H^{-1}\left(\frac{1}{\beta} - 1 + \frac{1}{\omega} H^{\omega}\right)\\
H &= \left[\left(\frac{1-\beta}{\beta}\right)\left(\frac{\omega}{\omega-1}\right)\right]^{\frac{1}{\omega}} && (28)
\end{aligned}
$$

GHH

- From (23) we get

$$N^{\alpha+\theta} = (1-\alpha)K^{\alpha}H^{\alpha} \tag{29}$$

From (24) we get

$$N^{1-\alpha} \quad = \quad \frac{H^{\omega-1-\alpha+1}K^{1-\alpha}}{\alpha} \tag{30}$$

From (29) we get

$$N = (1-\alpha)^{\frac{1}{\alpha+\theta}}K^{\frac{\alpha}{\alpha+\theta}}H^{\frac{\alpha}{\alpha+\theta}} \tag{31}$$

Plug (31) in (30) to get

$$(1-\alpha)^{\frac{1-\alpha}{\alpha+\theta}}K^{\frac{\alpha(1-\alpha)}{\alpha+\theta}}H^{\frac{\alpha(1-\alpha)}{\alpha+\theta}} = \frac{H^{\omega-\alpha}K^{1-\alpha}}{\alpha} \tag{32}$$

GHH

- Solving equation (32) to get the steady state capital stock K as

$$K = (1-\alpha)^{\frac{1}{\theta}} \alpha^{\frac{\alpha+\theta}{(1-\alpha)\theta}} H^{\frac{\alpha(1+\theta)-\omega(\alpha+\theta)}{\theta(1-\alpha)}} \tag{33}$$

Plug (33) in (29) to get the steady state level of labour services as

$$N = (1-\alpha)^{\frac{1}{\alpha+\theta}} K^{\frac{\alpha}{\alpha+\theta}} H^{\frac{\alpha}{\alpha+\theta}} \tag{34}$$

With the steady state expressions for H, K, and N in hand we can compute the steady state level of C from (26). You are now prepared to create your own DYNARE file to replicate this model!!!

GHH

- Examine impulse responses.
- A couple of points: try to incorporate this type of shock into the standard RBC we have examined
- You will get consumption to move countercyclically w.r.t to an IS shock.
- In the GHH model however consumption moves procyclically: variable capacity utilization is the reason
- An $\uparrow$ in cap u implies a higher marginal product of labour and this in turn a higher opportunity cost of leisure
- This creates intra-temporal substitution between leisure and consumption